ACFEL TR 62

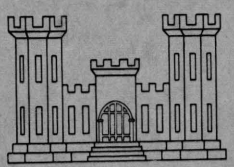
CORPS OF ENGINEERS, U.S. ARMY

DESIGN AND OPERATION OF AN HYDRAULIC ANALOG COMPUTER FOR STUDIES OF FREEZING AND THAWING OF SOILS

by

Massachusetts Institute of Technology
Department of Civil and Sanitary Engineering
Soil Engineering Division

Contract No. DA-19-016-eng-2743



TECHNICAL REPORT NO. 62

UNDER CONTRACT
WITH

ARCTIC CONSTRUCTION AND FROST EFFECTS LABORATORY
NEW ENGLAND DIVISION
BOSTON, MASSACHUSETTS

FOR

OFFICE OF THE CHIEF OF ENGINEERS
AIRFIELDS BRANCH
ENGINEERING DIVISION
MILITARY CONSTRUCTION

May 1956

PREFACE

This report is one of three reports describing results of investigations for the "Design, Fabrication, Exploratory Operation and Operational Manual of a Hydraulic Analog Computer". The investigation has been sponsored by the Arctic Construction and Frost Effects Laboratory, New England Division, Corps of Engineers, U. S. Army, under contract No. DA-19-016-Eng-2743 with the Massachusetts Institute of Technology through its Division of Industrial Cooperation under DIC Project No. 5-7155.

The entire investigation is directed toward improvement of techniques for predicting subsurface temperatures, particularly with reference to frost and thaw penetration below airfield pavements. This report presents the design, construction, and operation of the hydraulic analog computer. Two other reports are: (a) Heat Transfer at the Air-Ground Interface with Special Reference to Airfield Pavements, and (b) Frost Penetration in Multilayer Soil Profiles.

TABLE OF CONTENTS

		Page No.
	Preface	iii
	List of Figures	vii
	Table of Notation	ix
	INTRODUCTION	
	Contract	1
	Scope	3
	M.I.T. Personnel	3
	Chapter 1	
	GENERAL	
1-01	Description of Computer and Analogous Relationships	5
1-02	Historical Developments	7
	Chapter 2	
	DESIGN OF COMPUTER	
2-01	Design Requirements	11
2-02	Working Fluid	12
	Chapter 3	
	DESIGN AND CONSTRUCTION OF COMPONENTS	
3-01	Hydraulic Resistors	17
3-02	Latent Heat Wells	22
3-03	Volumetric Heat Standpipes	24
3-04	The Programmer	25

TABLE OF CONTENTS (Contd.)

Page No.

Chapter 3 (contd.)

3-05 Connections, Stopcocks and Frame 27

Chapter 4

PROGRAMMING THE COMPUTER

4-01 The Scaling Procedure 29

4-02 The Computer Setup and Operation 33

4-03 Presenting the Computer Results 35

4-04 Verification of Computer Results 35

BIBLIOGRAPHY

Appendix A Upper Limits of Laminar Flow in Capillaries

Appendix B Calibration of Hydraulic Resistors

LIST OF FIGURES

Figure No.	Title
1.	Hydraulic Analog to Frost Penetration
2.	Photograph of Hydraulic Analog Computer
3.	Photograph of Programmer
4.	Photograph of Computer Components, Rear View
5.	Photograph of Resistor Type A and Latent Heat Well
6.	Programmer Frame, Working Drawing
7.	Latent Heat Well and Kymograph Additions, Working Drawing
8.	Resistors and Programmer Reservoir, Working Drawing
9.	Programmer Circuit Diagram
10.	Typical Performance of Latent Heat Well
11.	Latent Heat Well Calibration Chart
12.	Kymograph Calibration Chart
13.	Scaling Relationships, Analog to Prototype
14.	Scaling Sheet, Sample Solution
15.	Observation Sheet, Sample Solution
16.	Depth of Frost Penetration, Sample Solution
Appendix A:	
A-1	Upper Limits of Laminar Flow Through Circular Tubes and Between Parallel Plates
Appendix B:	
B-1	Schematic Drawings of Resistance Calibration Methods
B-2	Example of Resistor Calibration Charts

TABLE OF NOTATION

Symbol	Name	Usual Units
A	Combined cross-sectional area of standpipe and manometer tube	sq. in.
C	Volumetric heat	Btu per (cu.ft.)(deg. F.)
$\overline{C}$	Volumetric heat for a soil "lump" of unit area and thickness Th	Btu per deg.F.
d	Distance between centers of adjacent lumps	ft.
h	Analog head	in.
k	Thermal conductivity	Btu per (hr.)(ft.)(deg.F.)
L	Latent heat	Btu per cu. ft.
$\overline{L}$	Latent heat for a soil "lump" of unit area and thickness Th	Btu
Q	Rate of flow	cu. in. per min.
q	Rate of heat flow per unit area	Btu per (hr.) (sq. ft.)
R_a	Resistance of analog resistor	min. per sq. in.
R_p	Thermal resistance between centers of adjacent lumps	(deg.F.) (hr.) per Btu
T_s	Time scale	min. per hr.
Th	Thickness of a soil lump	ft.
t_a	Analog time	min.
t_p	Time in nature	hr.
U_s	Thermal energy scale	cu. in. per Btu
V	Stored water volume	cu. in.
V_L	Latent heat well volume	cu. in.
V_s	Temperature scale	in. per deg. F.
$\overline{u}$	Thermal energy in a soil lump	Btu
v	Temperature	deg. F.
x	Abscissa of surface temperature curve	in.
y	Ordinate of surface temperature curve	in.

INTRODUCTION

Contract. -

On 2 October, 1953, the Massachusetts Institute of Technology through its Division of Industrial Cooperation entered on a cost-type research contract with the Arctic Construction and Frost Effects Laboratory, New England Division, Corps of Engineers, U. S. Army. This contract, No. DA-19-016-Eng-2743, is entitled "Design, Fabrication, Exploratory Operation and Operational Manual of a Hydraulic Analog Computer". This is one of three reports covering investigations at M.I.T. during the first two years of this contract.

Specifically, this report describes results of investigations called for in the following paragraphs of Appendix A of the contract:

"1. The project for the first year will consist of making a detailed design, preparing working drawings, and constructing a hydraulic analog computer with the following principal features:

(a) The hydraulic analog computer shall consist of a series of small vertical tubes or wells connected to each other at the bottom through bundles of capillary tubes or porous stones. Each vertical tube shall represent an element of soil at a given distance below the ground surface. The surface of a liquid in a tube shall represent a temperature if the problem is a thermal one, or a head, if a hydraulic problem. The capillary tubes or porous stones shall be analogous to the thermal or hydraulic resistance to flow. To simulate latent heat for freezing and thawing problems, an expansion well shall be provided at a temperature level representing the freezing point of soil moisture. The size of the well and the size of the vertical tubes shall be adjustable to account for variable latent heat and volumetric heat.

(b) The basic hydraulic analog computer shall consist of approximately 20 to 30 tubes and shall occupy a space about 3 feet by 6 feet in area. The computer shall be so designed and constructed to permit extension in length without limit, if required, and shall be articulated in a series of boards which

may be operated either independently or in tandem.

(c) The hydraulic analog computer shall be a generally applicable analyzer for one-dimensional heat transfer problems both with and without the latent heat condition.

(d) The hydraulic analog computer shall readily take into account variations in soil properties such as changes in thermal conductivity, volumetric heat and latent heat with depth in the soil, and variation of conductivity and specific heat with soil temperature.

(3) The hydraulic analog computer shall readily take into account programmed surface temperature or loading fluctuations of any sort, including experimentally observed distributions.

2. After completion of construction of the basic computer, the contractor shall -

(a) Make necessary adjustments and modifications to place the hydraulic analog computer in satisfactory operating condition.

(b) Assemble the hydraulic analog computer at an area at Massachusetts Institute of Technology where the equipment can be used for the solutions of problems."

"5. During the course of the second years' investigation, the contractor shall prepare a manual of instructions for future operators and equipment, based on the information which has been obtained regarding the manipulation of the apparatus in the solution of the various problems encountered."

Scope. -

The scope of this phase of the contract investigation is, in reality, essentially as specified under "Contract". The report is divided into four chapters.

Chapter 1 presents a general discussion of the computer and the analogous relationships among parameters in the thermal and fluid flow

systems. Further, historical developments in hydraulic analogs both in this country and abroad are presented.

Design requirements and selection of a working fluid comprise the major parts of Chapter 2 while the construction and assembly of the various components of the computer are discussed in Chapter 3.

Finally, the programming procedure for the hydraulic analog computer is presented in Chapter 4. A sample problem solution is given.

M.I.T. Personnel. -

Investigations under this contract have been carried out under the supervision of Harl P. Aldrich, Jr., Assistant Professor of Soil Mechanics, Department of Civil and Sanitary Engineering. During the first year, Henry M. Paynter, then Assistant Professor of Hydraulic Engineering, also supervised parts of the investigation.

Dr. Ronald F. Scott, Soils Engineer at the Arctic Construction and Frost Effects Laboratory, formerly Research Assistant in Civil Engineering, carried out the majority of the investigation described herein. Mr. Rasmus S. Nordal, formerly half-time Research Assistant in Civil Engineering, worked on the project during its second year. He contributed to the construction and operation of the computer.

Dr. Aldrich and Dr. Scott, assisted by Mr. George Leung and Mr. Nordal, are responsible for the technical writeup.

Chapter 1

GENERAL

The hydraulic analog computer has been designed primarily to solve complex one-dimensional thermal problems in soil involving freezing and thawing. Of particular interest is the determination of the thermal regime below airfield pavements and buildings constructed on non-uniform soil. More specifically, the computer will give the depth of freezing or thawing at any time under given soil conditions and boundary temperature conditions.

Before beginning a detailed description of the design, construction and operation of the hydraulic analog computer, it will be desirable to consider in a general way the manner in which the computer simulates the physical conditions of the problems it solves. Further, since other analogs have been built in the past, a fairly complete description of their construction is also included. The reader will find the remaining parts of the report much easier to follow after he reads the following general discussion.

1-01 Description of Computer and Analogous Relationships. -

Fundamental physical and mathematical principles which are involved in the one-dimensional thermal problem in soil are well established and have been reported elsewhere (1) (2)[1]. Mathematical solutions of the differential equations have been presented for various simplified cases. However, an analytical solution is not practically obtainable for typical cases involving non-uniform soil and/or time-varying surface temperature. Therefore, these complex problems must be solved by establishing a finite difference approximation to the fundamental differential equations and solving these numerically or with the aid of an analog. We shall describe the latter, in particular the hydraulic analog solution.

Figure 1 illustrates diagrammatically the analogous relationships in the thermal and fluid flow systems while Figure 2 is a photograph of

---

[1]Numbers in parenthesis refer to references in the bibliography.

the computer. As heat is removed from a given layer of soil its temperature falls to the freezing point of soil moisture, normally 32° F. The temperature dwells at this point until the latent heat of fusion of the soil moisture is removed or, in other words, until the soil moisture freezes. After the layer is frozen its temperature drops below freezing. A soil profile may be divided into such layers, each layer being represented in the computer by a glass tube standpipe with an expansion reservoir. Water level in a standpipe corresponds to temperature for the soil layer it represents. Flow of heat within the soil is represented in the computing device by the flow of water. The characteristic of each standpipe and adjustable expansion reservoir, then, is analogous to the thermal energy versus temperature characteristic of the soil.

To represent resistance to heat flow between adjacent soil layers, a resistance to fluid flow between standpipes must be provided. In the computer, laminar flow takes place in the hydraulic resistance through openings of capillary size which can be continuously varied in length.

Before an actual problem can be solved on the computer, the soil profile and its thermal properties must be known. Generally, the thermal conductivity, volumetric heat and latent heat of each layer may be estimated from the water content and dry unit weight of the soil. The computer may then be set up with the appropriate standpipes, latent heat reservoir openings, and hydraulic resistances. Ground surface temperature, represented by the water reservoir to the left in each figure, may be programmed automatically to follow actual or assumed variations in surface temperature with time by means of a moving paper templet and servo motor system. A typical time scale is, for example, the correspondence of one minute of computer operation to two days of thermal variations in nature.

The analog computer can handle such refinements as variation of volumetric heat and thermal conductivity with temperature. Furthermore, variations in the freezing temperature of soil moisture from layer to layer may be easily accounted for by raising or lowering latent heat wells. In fact, almost any complex one-dimensional diffusion problem can be solved on the computer. Thus consolidation problems in soil engineering and diffusion problems in electrical engineering and other fields may be studied as well as heat transfer situations.

The principal advantages of the computer are simplicity of programming and operation, and provision for a continual visual check on the solution. While the number of units in the apparatus can be increased indefinitely to give more refined solutions, the accuracy of the present 20 element computer has been clearly demonstrated, as described later, by solving problems with known analytic solutions.

1-02 Historical Developments. -

Apparently hydraulic analog computers originated almost simultaneously in the United States and in Russia in 1935 and 1936. Moore (3) built and patented an apparatus which he called a "Hydrocal" for the solution, principally, of heat flow problems through building walls. At about the same time Lukyanov (4) built a "Hydraulic Apparatus" in the Russian Institute of Roads and Construction which he used to study a variety of problems including freezing in soil. A brief description of each of the first two analogs is in order, as it will effectively demonstrate the principles involved.

Moore's early Hydrocal was extended and modified in 1939-1940. Dr. Scott saw the apparatus during a visit to the University of Michigan in 1953. It consisted of eighteen vertical standpipes and connecting resistances. Moore obtained a variable hydraulic resistance by an appropriate combination, in series and/or in parallel, of calibrated capillary tubes. He controlled the temperature of the working fluid which was water, by immersing all the resistance tubes in a constant temperature water bath.

Each "standpipe" in Moore's Hydrocal consisted, in reality, of two tubes; a fixed vertical manometer tube and a second larger tube which could be tilted back to different angles. Thus, the adjustable tube could be tilted, within limits, to give an effective cross-sectional area which, when combined with the manometer tube, represented the appropriate capacitance or volumetric heat. Standpipes were connected to a manifold which, with various valves, aided the programming and solution of problems. Surface temperature variations were programmed manually by raising or lowering a reservoir which moved in guides by pulleys.

The principal disadvantages of Moore's apparatus were that the complex "piping" needed to connect the capillary tubes admitted air

which was difficult to remove and rubber connecting tubes perished and some corrosion of brass fittings occurred. Moore was aware of the disadvantages of water as a working fluid (meniscus effects, corrosion, bacteria growth, etc.) and experimented with other liquids without much success. Results of some experiments carried out on the Hydrocal are described by Lamb (5).

The original Russian apparatus described by Lukyanov (4), was a good deal broader in its scope as well as in its dimensions. Further, Lukyanov states that two other devices were built at the beginning of World War II to accommodate two-dimensional flow problems with boundary conditions programmed automatically. The writers have found no publications which describe these analogs.

Lukyanov's analog incorporated variable hydraulic resistances which apparently consisted of capillary tubes of continuously variable length. While the vertical standpipes were of a constant cross-sectional area the effective area was varied by inserting prismatic rods. This approach leads to the possibility of accounting for variations in volumetric heat with temperature by inserting tapered rods. There is no evidence that Lukyanov actually did this but the possibility of the refinement is obvious.

While Moore's Hydrocal was not designed to account for heat stored or released on freezing or thawing, the Russian apparatus did incorporate this latent heat condition. For this purpose an ingenious device was attached to each standpipe whereby a hollow cylinder was connected to the standpipe by a rubber tube from its base. The cylinder was floated in water in such a way that it transferred its liquid volume (latent heat) to the standpipe while the fluid level in the standpipe remained constant (freezing condition).

It would appear, from dimensions given in the reference, that solutions on Lukyanov's apparatus took a long time. The operating fluid was not mentioned specifically but it was presumably water.

Several hydraulic analogs to diffusion problems have been built since 1945. In Sweden, Backstrom (6) (7) studied the cooling of bodies in refrigerating cabinets by means of various kinds of simple hydraulic analogies using capillary resistance tubes cut to appropriate size. He also had occasion to examine the freezing of bodies placed in a refrigerator.

He used a very simple latent heat device, applicable to one-directional heat flow, called a "chicken feed" which carefully meters a given quantity of water into the standpipes when the water level corresponding to freezing is reached.

In the United States in 1948, Leopold (8) (9) constructed a hydraulic analog for studying problems connected with thermal storage, radiation, convection and conduction. This apparatus is located at Pennsylvania State University where its use is continued. The resistance tubes are made of coiled metal tubing and must be replaced for each problem involving a different resistance. This inconvenience is not so serious in Leopold's work because the materials through which thermal conduction takes place are typical building materials having reasonably well established thermal conductivities. Originally the boundary temperature was programmed by hand but later a motor driven cam was introduced in order to simulate cyclic solar radiation and air temperature. Leopold substituted a silicone oil (Dow-Corning DC 20) for water as the working fluid.

In 1948, Barron (10) described a "Viscous Flow Tube Model" which was used to study one-dimensional consolidation and swelling in soil. He proposed a model design for the freezing or thawing problem where each standpipe had an enlargement of suitable dimensions to account for latent heat. When water level fell to the enlargement (freezing point) the tube was raised a distance equal to the height of the enlargement at a rate selected to keep the water elevation constant at the freezing point.

An air-flow analog computer was built by Coyle (11) in England in 1951. The air is contained in a closed circuit which also includes a transformer oil, of low viscosity and vapor pressure, which is used to indicate the pressure in the air. While air exhibits little variation in viscosity with temperature, it has the undesirable properties of having an extremely low viscosity and some compressibility. Coyle says the latter must be taken into account in accurate work. The resistance tubes are metal cupro-nickel and are very small indeed (0.017" bore) because of the low air viscosity. They cannot be adjusted continuously. All the joints in the analog must be made air-tight and, unlike a water leak, an air leak is very difficult to detect. The standpipes were designed to include a variation of specific heat with temperature since the problems involved heat conduction in metals where the specific heat and thermal

conductivity may vary appreciably with temperature.

At the Massachusetts Institute of Technology, a small hydraulic analog was constructed in 1949 (12) (13) to study and demonstrate consolidation problems in soil. Further, it was used in the settlement analysis of the Hayden Library of M.I.T. Again small glass capillary tubes were used to simulate the permeability of the soil.

A paper by Juhasz and Hooper (14) was published in 1953 which described a simple hydraulic analog. References to similar analogies have been made in articles by de Laclemandiere (15) and Ramachandran (16).

Chapter 2

DESIGN OF COMPUTER

This chapter sets forth the basic requirements which must be met in the design of the hydraulic analog computer. The selection of a working fluid is treated separately because of its important influence on the design. The actual design and construction of individual components of the computer, together with working drawings, is presented in the following chapter.

2-01 Design Requirements. -

Although the authors gained considerable knowledge of hydraulic analog computers from the experience of other investigators, it was deemed advisable to make the apparatus as simple and flexible as possible because of the research nature of the investigation. This flexibility would allow for solutions of other complex diffusion problems in addition to freezing and thawing in soil. Further, it was felt that simplicity would lead to real usefulness. Consequently, the major design requirements were established as follows:

(a) The cylindrical standpipe tubes representing volumetric heat, should be easily varied in diameter by inserting formers and/or by utilizing tubes of different diameters.

(b) The hydraulic resistors should be continuously adjustable, without having to remove the operating liquid, to accommodate the wide range of thermal conductivity found in soils and to enable adjustment during the operation of a problem in a case where conductivity varied with temperature.

(c) An improved and simplified latent heat simulator was necessary in which freezing and thawing could take place, without attention or adjustment of the element, when temperatures oscillated about the freezing point.

(d) If possible, water should not be used as a working fluid because of its viscosity variation with temperature and affinity for air. A thermostatically controlled temperature bath was to be avoided.

(e) The working fluid should be visible throughout the computer so that air bubbles and dirt could be easily detected. The components of the computer should, therefore, be transparent.

(f) Stopcocks for each element must be provided which could be operated easily and, if possible, simultaneously (in gangs).

(g) No part of the apparatus should be susceptible to corrosion by the working fluid or outside agencies.

(h) The fluid meniscus in the standpipe should be eliminated because of the difficulty of reading the fluid level and because of its small effect on the water movement.

(i) A programming apparatus which can be operated automatically or by hand, should be incorporated for the surface temperature variation.

(j) The computer should consist of about 20 elements or units to cover the required range of one-dimensional problems.

(k) The total space required should not exceed 6 or 7 feet by 3 feet in area.

Attention was initially concentrated on the resistance tubes which are the most important part of the apparatus. However, before the dimensions of the capillary tube through which the operating fluid would flow could be established, it was necessary to select a working fluid and to determine its physical and chemical properties.

2-02 Working Fluid. -

After an extensive investigation of a variety of possible working fluids, it was found that water did indeed come closest to fulfilling all the desirable requirements. Factors leading to the selection of water are outlined in the following paragraphs after the desirable requirements of the working fluid have been discussed further.

Design requirement (b) states that the hydraulic resistor should be continuously variable without having to remove and replace the working fluid. Thus, once the resistor has been placed in operation the working fluid may remain in contact with the resistor for long periods of time, until the resistor is removed to make way for another or for cleaning purposes. Therefore, the fluid must neither corrode the resistor (or the remainder of the apparatus) nor deposit solid or gummy films on the

resistor passages. Since the dimensions of the passages, even for a relatively viscous fluid, must be small to insure laminar flow, this places severe limitations on both the fluid and material of which the resistor is constructed.

The necessity of excluding air from the apparatus, in particular from the hydraulic resistors, placed additional requirements on the working fluid. Since the resistor may have a somewhat complicated internal shape, it appeared at an early stage that it might be necessary to introduce the liquid into the resistor under a vacuum to insure the absence of air bubbles. In any event, any simple saturation procedure could require large quantities of the fluid, some of which may spill from the apparatus. Therefore the fluid should not be expensive or toxic and should not leave a residue on evaporation or be gummy or sticky or offensive in any way.

Since the apparatus would be exposed to air, although well shielded from turbulent contact, the working fluid should not dissolve quantities of one of the gases of the air which, under a varying operating pressure and temperature, could come out of solution in one of the capillary passages and restrict the fluid flow in a manner not allowed for in calculations. Dust caps were proposed for the volumetric heat standpipes to limit the entry of foreign matter which might also find its way into a resistor capillary.

During the solution of a diffusion problem, fluid levels in standpipes rise and fall and latent heat wells may fill and empty. The working fluid must, therefore, drain cleanly from walls of capacitance tubes and it must fill and empty the latent heat wells entirely. This implies a non surface wetting liquid which is also desirable for reading liquid levels accurately because the contact angle between the liquid and wall of the standpipes becomes 90°, eliminating a meniscus.

Since the analog computer will be operated in a laboratory with no refined temperature control, it would be desirable to have a liquid which has no viscosity variation with temperature over the range of say 20 to 30°C. Use of such a fluid would eliminate having to immerse the hydraulic resistors in a constant temperature bath in order to insure a constant resistance during the solution of a diffusion problem.

Now that the desirable characteristics of a working fluid have

been discussed, a list will be given of all the fluids which were considered, together with their principal advantages and disadvantages. No extended evaluation of each is given, since its possible utility is, in most cases, simply disposed of:

Glycerol:

Advantages: Does not dissolve air in significant amounts. Has a high viscosity and is easily miscible with water to give any required viscosity within a large range, which can be maintained however, only if the water/glycerol proportions remain constant.

Disadvantages: Has a high variation of viscosity with temperature at all water/glycerol ratios. Glass is wetted by it.

Carbon Tetrachloride:

Advantages: Does not entrain air and has a fairly low variation of viscosity with temperature.

Disadvantages: Evaporates readily and is slightly toxic. Has a low viscosity and high density. Is a good solvent.

"Cellosolve", "Carbitol" and other industrial organic solvents:*

Advantages: Have relatively high viscosities which can be varied by mixing with water. Do not dissolve and release air.

Disadvantages: Are strongly solvent and usually have a fairly high evaporation rate. They wet glass.

Kerosene and other fuel oils:

Advantages: Possess no air-entraining properties and have relatively high viscosities.

Disadvantages: Are inflammable and their viscosity varies considerably with temperature. They vaporize easily.

Silicone oils:

Advantages: Are stable and non-poisonous. Have a low variation of viscosity with temperature. Have a relatively high viscosity.

*Manufactured by Carbide and Carbon Chemicals Company, a Division of Union Carbide and Carbon Corporation.

Disadvantages: Are very expensive, so that any leakage or waste would have to be avoided. They wet glass.

Finally, attention turned back to water which was selected for use in the computer. Water dissolves air to a small extent (17) and tends to release this air under changed temperature and pressure conditions which is not desirable. An investigation was made to see if any additives were known which would eliminate this undesirable property. It was found that an almost saturated solution of Epsom salts in water did not absorb air to the same degree as pure water. However, the difficulties incumbent upon using such a solution in the proposed apparatus were obvious and were too great to be overcome. Consequently, it was proposed to use distilled de-aired water and to protect the water from turbulent contact with the air.

Water has a rather low viscosity and thus the dimensions of the resistor capillary must be small to insure laminar flow. This in turn leads to smaller allowable tolerances in the construction of the resistors.

Clean water wets clean glass but this disadvantage can be avoided by the use of a commercial product called Drifilm.*

The viscosity of water varies with temperature but to a smaller degree than most other fluids, except, the silicone oils. It was found that if the computer was placed in a normal laboratory room, away from drafts and radiators, it was not likely to undergo large temperature changes (of the order of 0.2 to 0.5°C) in the course of an experiment.

A final precaution was taken to limit the possible growth of bacteria in the water by adding 1/100 of one per cent of mercuric chloride. There is no possibility of this quantity of the chemical crystallizing in the system and blocking the flow.

In conclusion, distilled de-aired water has been found to be a satisfactory working fluid although it does not possess all of the desirable requirements stated originally.

---

* Drifilm 9987, General Electric Organosilicon Products.

Chapter 3

DESIGN AND CONSTRUCTION OF COMPONENTS

The hydraulic analog computer, designed and constructed under this contract, contains four basic elements, each of which must be designed and constructed to meet certain requirements. These elements are the hydraulic resistors, latent heat wells, volumetric heat standpipes, and the programming device. Each of these elements will be discussed in turn after which the connections, stopcocks and frame for the computer will be described.

3-01 Hydraulic Resistors. -

Two entirely different hydraulic resistors have been used with varying degrees of success in the analog computer. The first is shown in Figures 5 and 8 and the second in Figure 8.

a. Design Requirements. -

The hydraulic resistance R_a, is defined by the following expression:

$$Q = \frac{1}{R_a} (h_1 - h_2)$$

where:

Q, rate of flow in cu. in. per min.

R_a, hydraulic resistance in min. per sq. in.

$h_1 - h_2$, head difference in in.

In the analog, R_a must not vary with the head difference or, in other words, the rate of flow or velocity must be directly proportional to the head difference. This analog requirement, which follows from the proportionality between heat flow and thermal gradient, implies laminar flow. The upper limits of laminar flow in capillaries are discussed in Appendix A and experimental procedures for determining the resistance R_a, are given in Appendix B.

The principal design requirement involved the selection of a

capillary cross-section, which could be easily varied in length, in which laminar flow would occur under the anticipated maximum head. Since volumetric heat standpipes were to be about 20 inches long, it would first appear that resistors should be designed for this condition. In reality however, the head difference seldom exceeds 1/2 to 1 inch except for the standpipe at the upper boundary in problems involving a step change in surface temperature. Even so, in a typical problem having a solution time from 5 to 10 hours, a sudden change in head can be adequately represented by lowering the boundary reservoir over a period of several minutes. Therefore, the resistor capillary was designed to maintain laminar flow under head differences of 1 inch. Considerations presented in Appendix A indicated that the capillary should have a cross-sectional area of between 0.001 and 0.002 sq. in.

Loss of head at the entrance and exit of the capillary should be kept as small as possible. The importance of these losses which are proportional to the square of the velocity, depends on their magnitude relative to the head loss in laminar flow through the resistor capillary.

The hydraulic resistors should be constructed if possible, of a transparent material so that air could be detected visually. Resistor Type A fulfilled this requirement, having been constructed of lucite, but resistor Type B was built of stainless steel.

It was anticipated that each resistor would have a different calibration curve even though a single calibration for all resistors would be desirable. Each resistor should have a graduated scale so that it can be easily set to a given resistance.

b. Hydraulic Resistor Type A. -

The hydraulic resistor used during the first year of computer operation is called Resistor Type A and is shown in Figures 5 and 8. The resistance range is approximately 3.0 to 80.0 min. per sq. in.

Resistor A was designed along the lines of an electrical helical resistance. Water flows through a small rectangular space which is formed between square threads of a core which is screwed into a barrel. In operation, water flows into the resistor chamber at one end, around the core in a helical groove until it reaches an exit hole in the side of the barrel. The flow is, indeed, reversible. The length of the capillary can

be varied by screwing the core within the barrel. The unit is constructed of lucite and sealed at the ends by either rubber stoppers or threaded lucite caps with 0-ring bushings.

A graduated scale calibrated in number of turns is attached to the barrel. A removable graduated drum made of aluminum serves as a useful "micrometer" for setting the core position. The graduated drum can be readily transferred from one resistor to another.

The total quantity of water in the resistor remains constant regardless of the setting. Thus, if the resistance is altered during the solution of a problem no sources or sinks are introduced. Before the core is rotated the bypass valve must be opened to enable water to pass freely from one end of the resistor to the other and thus to prevent a pressure difference between the ends.

One of the problems in the initial assembly of the resistors is their saturation with water. Several methods of accomplishing this were attempted. After trying to evacuate all the air from a resistor before filling it with water, which proved only moderately successful, one final procedure resolved itself into completely immersing both the core and the barrel in distilled de-aired water, and assembling the resistor below the surface. Another method involved holding the resistor below the level of the computer reservoir, which was connected to the bottom of the resistor, and allowing water to flow through the resistor continuously while assembling it. In this way the presence of water in all the passages was assured before the final connections were made. When air did occur in a resistor capillary, it was possible to apply an excessive pressure difference across the resistor, by blowing down a vertical standpipe, and to accelerate the water in the capillary to a point where the air bubbles were removed. Any small amount of air which accumulated in parts of the resistor where the water was stagnant was not found to be of large enough volume to affect the solutions materially.

The resistance of a pipe of given diameter and length which is curved (i.e. has the form of a helix, and through which steady state flow is taking place, is different from that of a straight pipe under the same conditions. This difference, presumably arising because of the helical movement of fluid relative to the pipe itself which leads to a greater energy loss, has been investigated by others and described in the

literature. The importance of this error on the performance of Resistor A was investigated. It was found that the ratio of the diameter of the core to the diameter of the capillary was too large for a significant difference in the friction factor to manifest itself.

A "pilot" apparatus consisting of six hydraulic resistors of the type described was constructed initially. These units were placed in operation and their performance was checked frequently. Two items which were of particular interest were the determination of the limiting conditions for laminar flow and the general stability of the resistor calibration. Procedures for determining the hydraulic resistance experimentally are presented in Appendix B. Since the laminar resistance varies directly with the length of the flow path, the calibration curve for the resistor should be linear.

Perhaps the first criterion is to determine the head difference at which laminar flow in the resistor breaks down, so that precautions can be taken to prevent this situation in a computer solution. If laminar flow exists at a reasonably high head under a low resistor setting (short path), it follows that flow at higher resistances must also be laminar. Therefore, a resistor was tested at two turns.

For the resistor having a capillary opening of 0.03 in. by 0.05 in., it was found that the rate of flow was indeed proportional to the head difference in the range in which the apparatus was to operate. A second resistor with a capillary opening 0.06 in. by 0.10 in. was built, but on testing it was found that it did not exhibit laminar flow at required heads.

Results of tests showed that the resistance at any one setting did not vary with the direction of flow. However, it was discovered that when a valve was adjusted to one setting at which its resistance was measured and then moved away from it and returned, the new resistance was not identical to the previous one. The differences were not large and are probably ascribable to the lateral movement of the core of the resistor making the capillary path of smaller cross-section on one side and larger on the other.

Further resistance determinations were made after an interval of a few days had elapsed, during which the resistor remained at one setting, and it was found that no observable change in the resistance had occurred. However, later tests have shown small discrepancies in the

direction of a slightly larger resistance with time. It was found by continual checking of the resistance at one setting, after moving the core away from that position and back again, that after a period of time the resistance seemed to settle down and remain fairly constant. This may be due to a gradual elimination of machining irregularities in the capillary boundaries or to the gradual formation of an absorbed water film on the lucite surface.

From the series of tests and study of the calibration charts which began to accumulate, it was concluded that, for very accurate analog solutions it would be necessary to set each resistor experimentally to the required resistance, immediately before running the test, and to check it at the end of the test. However, for tests involving rough estimates of temperature penetration into the soil, it would be sufficient to set each resistor according to values obtained from a calibration chart.

An additional problem encountered with Resistor A was that after a long period of time the resistor core began to stick within the barrel making the resistance adjustment difficult or impossible. The only practical method of remedying this fault, which seemed to be due to small distortions or swelling in the lucite, was to take the valve apart, determine if possible where the sticking occurred, and remove small shavings by machining until the valve operated easily once more. In addition to this the valves were greased very lightly with silicone stopcock grease before reassembly. In one or two cases the remachining was too extensive and water apparently moved across the threads in addition to flowing in the prescribed path.

These important disadvantages of Resistor A were the principal reasons for seeking a more acceptable design.

c. Hydraulic Resistor Type B. -

Hydraulic Resistor B was designed and constructed during the second year of computer operation. The final design is shown in Figure 8. Earlier designs leading to this resistor are described in Reference 20. The resistance range for Resistor B is approximately 0.5 to 7 min. per sq. in.

Resistor B consists simply of a hexagonal stainless steel plunger moved within a 5/16" O.D. stainless steel tube. Water flows through six

capillaries which are varied in length by sliding the plunger within the tube. The plunger, which is streamlined at the tip, has a maximum diameter which is slightly smaller than the inside diameter of the tube. Unlike Resistor A, a change in resistor setting does act like a source or sink. Thus, a minor error is introduced when the setting is changed during the course of a problem solution.

In order to guide the plunger and prevent rotation within the tube which may result in a change of resistance, the hexagonal rod is first flattened on 4 of 6 faces so as to fit more precisely the inside of the tube. Three pairs of indentations are then made with a center punch into the tube wall where the plunger enters it. These provide support and prevent rotation of the plunger.

On calibrating Resistor B it was found that its resistance at 25°C was approximately 0.50 min. per sq. in. per inch length of plunger within the tube, Figure B-2. The resistor was tested at a low setting (R_a = 0.54 min. per sq. in.) and found to exhibit laminar flow to a maximum head h_1 of 2 inches which is represented in Figure B-2 by a head ratio of about 3. These results were considered satisfactory and 10 resistors of this type were constructed.

In the solution of analog problems, it was found that the calibration of this resistor did not change appreciably with time. Thus, setting the desired resistance from a calibration curve proved to be entirely satisfactory. The principal problem appears, in fact, to be related to the accuracy with which the length of plunger inserted in the tube can be measured.

At the present time, the analog computer is assembled with the first ten resistors of this type. Resistors of Type A are being used, where necessary, to give the larger resistances required for thicker soil layers at greater depths below ground surface.

3-02 Latent Heat Wells. -

A fluid reservoir selected to simulate the stored latent heat of fusion of soil moisture in a layer of soil is called the latent heat well. The final design adopted for construction is shown in Figures 5 and 7. Figure 4 is a photograph showing assembled latent heat wells in operating position on the computer.

The latent heat well consists of two transparent lucite plates separated by a distance of 3/8 inches and centered on the freezing point level in the vertical volumetric heat standpipe. The cross-sectional area of the well is 0.656 sq. in. The volume of the well is varied by a rectangular lucite slide which moves between the outer plates. The usable volume can be varied from zero to approximately 6 cu. in.

While this design appears extremely simple, it required considerable experimental testing before an optimum design was selected. Considerations leading to the design follow.

It was first decided that none of the latent heat reservoirs described in Chapter 1 would be suitable for the present analog largely because of their complexity and lack of flexibility.

While the soil ideally yields all its latent heat at 32° F when freezing, in reality freezing may take place over a finite temperature range at or below 32° F. Therefore, it was felt that a slight approximation could be allowed in the latent heat well in the form of allowing the water level to vary slightly while the well emptied or filled. In the final design, "freezing" takes place over a temperature range of about 2° F since a typical temperature scale is 0.2 in. equals 1.0° F. The performance of the well on draining and filling is shown in Figure 10 while the ideal performance and analogous ideal thermal energy diagram for soil are shown in Figure 13.

An important design requirement is that the well must fill completely without attention and without trapping air and it must also drain freely and completely. In the final design, a tapered groove is provided, Figure 7, in the center of the top lucite plate to allow air, which might otherwise be trapped at the rear of the well, to escape. To discourage water from entering the groove before the air escapes, the slot is coated with a silicone non-wetting agent, Drifilm 9987. To aid the exit of water from the well when the water level is falling, two grooves were cut along the sides of the bottom lucite plate. Short pieces of cotton wick were inserted to hang into the vertical standpipe slightly below the level of the latent heat well. These wicks serve to "siphon" water which tends to remain along the sides and corners of the reservoir while the water drains very slowly.

At the back of the well, a slide is provided for adjusting the

capacity of the chamber to accommodate different values of latent heat. Since this slide and the well are of a rectangular shape, it was not easy to achieve a satisfactory seal at the rear end of the well while permitting the slide to be adjusted easily. A solution was obtained by cutting V-shaped grooves in the back of the well around the rectangular opening and providing matching grooves in a small capping piece of lucite which screwed onto the back. A piece of graphite cord was stuffed into the groove to serve as a seal. In order to adjust the well setting, it is necessary to slack the four cap screws slightly, move the slide the required amount, and tighten the screws again to maintain the seal. The movable slide is provided with a semi-circular notch at the front end. Thus, when the slide is closed (zero latent heat) the volumetric heat tube is continuous through the freezing temperature. For cases when the area of volumetric heat standpipes does not match the area of the semi-circular notch, the latent heat well may be removed entirely and a single long standpipe used.

Latent heat wells are provided for the first 12 elements of the computer only. The remaining 8 elements are used to establish the lower boundary condition in a problem solution.

3-03 Volumetric Heat Standpipes. -

Vertical glass tubes about 10 inches long are used to represent the volumetric heat capacity of the soil. Tubes of varying diameter (typical size, 9/32 in. I.D.), are provided with threaded lucite caps to enable the tube to be attached easily to either side of the latent heat wells. A seal is accomplished by means of an 0-ring. Loose dust caps, covered with wire gauze, are placed on top of the standpipes to prevent contamination of the water. All standpipes are coated on the inside with Drifilm 9987, the non-wetting agent, to assist in the reading of the water level and to minimize errors due to the meniscus. The volumetric heat standpipes are shown in Figures 2 and 7.

In earlier designs, the volumetric heat standpipes were to be cemented to the latent heat wells halfway along their 20 inch length. Since this required the cutting of a hole and slots in the pipe, it was necessary to consider the use of plastic tubing for the purpose. Further, plastic tubing was considered to be less breakable. Lucite tubing was thus selected.

Lucite tubing of small diameter is extruded and is highly variable in diameter which necessitated careful selection to obtain uniform bore. When suitable lengths were found they were connected to the latent heat wells as indicated above. At this time, and on various other occasions during the construction and use of the apparatus, trouble was experienced with cracking of the thin lucite tubes, beginning at a stressed point, and extending on occasion several inches along the length of the tube in a helical fashion. It was discovered that this was partly due to residual stresses in the tube due to insufficient annealing before and after machining and attachment to screw caps, and partly to solvent migration from those sections of the tube which had been cemented to other parts of the apparatus. The cracking proved so troublesome that it eventually led to the replacement of the plastic tube with glass.

A manometer tube of small diameter is attached to each volumetric heat standpipe for the sole purpose of enabling accurate water level readings at temperatures around the freezing point. The stored water in this manometer tube must be accounted for in the problem scaling as will be shown later.

3-04 The Programmer. -

The design of a device for programming automatically the reservoir water level to represent any desired surface temperature presented one of the most troublesome problems in the computer design. Figure 3 is a photograph of the programming device and Figures 6, 7, 8 and 9 include working drawings of the component parts.

We seek to transfer a curve of temperature versus time into corresponding water elevations in a reservoir. Since the temperature curve may contain steep slopes, a mechanical arrangement involving a stiff former with a cam resting on its surface was not considered to be desirable. Therefore, it was evident from the start that a reversible electric motor would be required to operate the reservoir. No difficulty was experienced in obtaining a suitable motor and setting up a circuit to operate it in either direction. The principal difficulty arose in devising an arrangement to inform the motor what to do.

After considerable study and several trial designs, it was decided that the temperature curve to be programmed would be plotted to an

appropriate scale on a piece of firm paper such as commercial wrapping paper or heavy weight graph paper. The paper would be cut along the curve and wrapped around a two drum kymograph which was set at the desired speed. The reservoir water level was programmed to follow this curve through an electrical probe which was rigidly attached to a float indicating the reservoir water level. The probe straddles the paper thus interfering with two sets of contact points in the probe. Through two small relays, these contacts operate the electric motor to raise the reservoir when no contact is made on either set of terminals, and to lower it when both contacts are made. The motor stops when one set of points only is in contact.

It is seen that reservoir water level follows the temperature curve by a series of finite steps. The precision of this programming depends on the spacing of the contact points, friction in the gearing, inertia of the system, and the speed and slope of the paper. When the reservoir level was held constant, it was found that as water flows to or from the reservoir it moved about 0.05 in. with each switching operation.

It is obvious that the reservoir itself does not follow the prescribed temperature curve because water level is continually changing relative to the reservoir. A reservoir in which water level is maintained at the same relative position to it, for example by means of an overflow, would have been easier to program. However, it would require a large quantity of distilled de-aired water among other things. The computer as constructed is a closed system, i.e. no water is removed or added once a problem solution begins.

Because the electrical circuits are such an important part of the working of the programmer, Figure 9 has been prepared to show the layout. Three circuits are involved. A step down from the 115V A.C. supply through a transformer and rectifier bridge to a 12.6V R.M.S. full wave rectified D.C. supply without smoothing, operates the two small relays through the probe contacts. Another circuit involves the relay contacts to operate the motor which requires a special circuit of its own to enable the reversing to take place. The motor is a 1/20 H.P. split-phase A.C. type, with a capacitor start and run, which can be operated in either direction by reversing the direction of current through one of the field coils.

Originally, the rapid switching of the motor, sometimes from forward to reverse and back, caused a great deal of trouble by continually blowing fuses. This may have been due in part to high transient currents which the operations generated. A solution was finally obtained by redesigning the probe, bending the motor relay contacts farther apart to minimize the chances of one set making contact before another had broken, and inserting small capacitors across the contacts to reduce the amount of sparking. It is not known to what extent each of these factors contributed to the final elimination of trouble.

3-05 Connections, Stopcocks, and Frame. -

Photographs given as Figures 2 and 4 show the component parts necessary to connect and control the hydraulic resistors, latent heat wells, volumetric heat standpipes, and programmer. Further, the frame supporting all components of the computer is indicated.

Because of the desirability of being able to examine the apparatus for air bubbles and since rubber deteriorates or becomes brittle with time, "Tygon" transparent flexible plastic tubing of 3/8" inside diameter was used for all flexible connections in the apparatus. All the connecting tubes were made of this large size to minimize head losses through all parts of the system outside the hydraulic resistors. No trouble has been experienced with the "Tygon" tubing in computer operations.

Where it was necessary to control or stop the flow of water between two adjacent standpipes and a resistor, a three-way lime glass stopcock was used which enabled flow to take place between any two openings or all three together. To allow the standpipes to be filled at the beginning of a problem or for calibration purposes, one manifold tube was constructed, running the full length of the apparatus. Entry into each standpipe is provided by means of an ordinary straight-through lime glass stopcock. All the stopcocks have been greased with silicone stopcock grease which is a very stable lubricant.

The components of the computer were first assembled on a frame consisting of 1/2 inch diameter aluminum rods connected by screw on angle pieces, Figure 4, which gave a very flexible assembly. This proved so convenient that it has been used throughout the final apparatus. It provides a light-weight, easily movable piece of equipment. New

attachments and extra units can be added with very little trouble.

Since the presence of the latent heat well at the freezing point makes it difficult to read the water level for about 1/2 inch above and below, an additional tube has been provided beside each standpipe to act as a manometer for reading water levels. Behind the manometer is a strip of card with two scales: 1) a convenient, frequently used scale containing 5 Fahrenheit degrees to the inch and having the 32° F mark centered on the latent heat well, and 2) an ordinary scale in inches and tenths of an inch reading from an arbitrary zero point at the bottom of the standpipe. Since a number of these manometer scales were required, it was found convenient to have one very carefully and accurately drawn. Photostats were made in the number required. The strips have been cemented to 3/4" wide 0.020" thick aluminum strips. They were then sprayed with a clear plastic spray for permanency before mounting.

The manometer tubes act with the volumetric heat standpipes in storing "heat". Therefore, they must be considered as one unit when scaling a problem for solution on the computer.

Chapter 4

PROGRAMMING THE COMPUTER

Programming a problem on the hydraulic analog computer is defined to include the computational work of scaling the physical problem in analog terms, the setup of the computer according to the scaling, and the running of the computer to obtain the solution. This chapter will serve to describe the programming procedure for the solution of freezing and thawing problems in soil.

4-01 The Scaling Procedure. -

We seek first to establish relationships between the physical conditions of the thermal problem and its analog representation. There are a variety of ways in which this scaling procedure may be carried out correctly. The most efficient procedure depends on the type of problem, available analog equipment, and personal preference of the investigator. One procedure will be presented here, in a series of steps to aid the beginner, for programming a seasonal freezing or thawing problem in stratified soil. Reference will be made to a summary of the scaling relationships, Figure 13, and to the standard scaling sheet, Figure 14. These may be folded out for convenient reference.

Step 1 Data required for the solution are obtained and entered in columns 1 to 4 (Figure 14). They are:

(a) Soil profile with water content and dry density to a depth at least equal to the freezing or thawing depth and preferably to a depth of 30 to 40 feet or more. The soil profile is plotted to scale in column 1.

(b) Thermal properties of each stratum making up the soil profile as follows:

Thermal conductivity, k
Volumetric heat, C
Latent heat of fusion, L

For this example, we shall make no distinction between frozen and unfrozen values of k and C. Thermal properties may be obtained, for example, by procedures given in References 1 and 2.

Step 2 The maximum depth of frost or thaw penetration is estimated. The first 10 to 12 elements of the analog computer are assigned to represent this depth since 12 units of the present computer are equipped with latent heat wells. In general, the thickness of finite layers of soil, called "lumps", which are represented by elements on the computer, are chosen to increase in thickness with depth. Thus, those near the surface, where temperature variations and thermal gradients are large, are the smallest.

The thickness Th, of the first soil lump is often chosen together with the combined area of the first standpipe and manometer tube. This establishes a relationship which must be met between all standpipe areas and corresponding lump thicknesses. The volumetric heat for the lump is given the notation $\overline{C}$ where:

$$\overline{C} = C \cdot Th, \qquad \text{Btu per } ^\circ F. \text{ (unit area)}$$

If the lump contains two or more soil types then $\overline{C}$ is the sum of the products $C \cdot Th'$ making up the lump. The thickness of remaining lumps and corresponding areas A, must now satisfy the following equation:

$$Th_n = Th_1 \cdot \frac{C_1}{C_n} \cdot \frac{A_n}{A_1} \qquad \text{or} \qquad \frac{A}{\overline{C}} = \text{Constant}$$

Standpipe areas are recorded in column 8 for corresponding lumps which are numbered consecutively for convenience as shown in column 10. The thickness of lumps is given in column 9 and the distance between centers of adjacent lumps is recorded in column 11.

Step 3 A temperature scale V_s, is the relationship between the prototype temperature and the water elevation, or head, in standpipes:

$$V_s = \frac{h}{v}, \text{ in. per } ^\circ F.$$

There are three factors to be taken into account when choosing the temperature scale. First, the variation in head that simulates the maximum temperature variation must be within the height of the standpipes and the range of reservoir movement. Second, the scale should be large enough to secure adequate precision of the observations. Third, since the

selection of V_s fixes the relationship between thermal energy and volume of water, the available volume of the latent heat wells sets an upper limit to the temperature scale. A typical temperature scale is 0.2 in. per °F.

When the temperature scale is chosen, the thermal energy scale U_s, becomes:

$$U_s = \frac{A}{\overline{C}} \cdot V_s, \text{ cu. in. per Btu}$$

where the ratio $A/\overline{C}$ is constant for all lumps.

The temperature and thermal energy scales are recorded at the top of the scaling sheet, Figure 14.

Step 4 The latent heat of each lump $\overline{L}$ in Btu (unit area), computed in the same manner as $\overline{C}$, is entered in column 7. The required latent heat well volume V_L, is then:

$$V_L = U_s \overline{L}, \text{ cu. in.}$$

which is entered in column 6. The slide opening required to give this volume is obtained from a calibration curve for the latent heat well, Figure 11, and recorded in column 5.

Step 5 A time scale T_s, which is the ratio of analog time to time in nature, is selected:

$$T_s = \frac{t_a}{t_p}, \text{ min. per hr.}$$

and recorded at the top of the scaling sheet. A scale which has been found convenient is 0.0208 min. per hr. (0.5 min. per day). The time scale fixes the value of hydraulic resistance for each resistor, as shown in the following steps, and must therefore, be selected accordingly.

Step 6 The thermal resistance R_p, between centers of adjacent lumps, distance d, is computed from:

$$R_p = \frac{d}{k}, \qquad (°F)\,(hr) \text{ per Btu}$$

which follows from writing Fourier's Law:

$$q = ki = k\frac{v_1 - v_2}{d} = \frac{v_1 - v_2}{R_p}$$

If two or more soil types occur between adjacent lump centers, the thermal resistance is computed for each layer. The sum of these resistances equals R_p, values of which are recorded in column 12.

Step 7 The corresponding values of analog resistance R_a, are now determined from:

$$R_a = T_s \frac{\overline{C}}{A} R_p, \text{ min. per sq. in.}$$

where the term $T_s \frac{\overline{C}}{A}$ is a constant for the scaling. Values of R_a are entered in column 13. From calibration curves for resistors, the resistor setting is determined for the room temperature at which the solution will be run, and is recorded in column 14.

The theory behind the analog solution, based on a finite difference approximation to the differential equations, is more thoroughly treated in References 10 and 21.

Subsurface temperatures vary in accordance with boundary temperature variations at ground surface and at some selected depth below the surface. These boundary conditions must be accounted for in the analog representation.

(a) Upper boundary condition. - The assumed or observed surface temperature curve constitutes the upper boundary condition. The surface temperature programming device, described in an earlier section, is set up to program the end reservoir in the analog to follow the required surface temperature curve.

The surface temperature curve must be drawn to coordinate scales in accordance with the temperature scale V_s, the time scale T_s, and the available kymograph speeds. The ordinate y in inches becomes:

$$y = V_s \cdot v_s$$

where v_s is the surface temperature in degrees F. The abscissa x in inches is related to the time in nature t_p, by:

$$x = T_s \cdot Sp \cdot t_p$$

where Sp is the kymograph speed in in. per min. Available kymograph speeds are given in Figure 12.

(b) Lower boundary condition. - At a sufficiently great depth below ground surface the temperature remains nearly constant during the year. This depth is known as the level of zero annual amplitude. The thermal gradient at this point and below is called the geothermal gradient. These factors constitute the lower boundary condition which must be accounted for in the analog solution.

The last 8 elements of the analog are provided to account for the lower boundary condition. They represent that part of the soil profile that is below the zone of freezing. Since the thermal gradients are small in this region, we are justified in using larger lumps. Frequently the lump spacing is doubled with each increment so it is possible to represent adequately a soil profile 50 to 100 feet in depth if necessary.

The geothermal gradient can readily be taken care of by adjusting initially the lower water levels to proper elevations. The capacitances of the lower lumps are so large that little change in water level occurs during a solution. If necessary, a metered quantity of water may be added to the last reservoir to represent the heat supplied from the earth's interior by the geothermal gradient.

4-02 The Computer Setup and Operation. -

Once the scaling calculations have been carried out for a problem and tabulated, it is necessary to adjust the elements of the computer to the values indicated by the calculations. Although this is not difficult, certain techniques have been derived because of the special characteristics of the computer, to enable the programming to be done as rapidly as convenient. The elements to be adjusted are the latent heat wells, the volumetric heat standpipes, the hydraulic resistors and the programmer. It is generally advisable to set the hydraulic resistors just before the problem is run in order to minimize possible changes due to temperature errors, etc.

The correct volume for latent heat wells is set from calibration curves. The movable slide is adjusted according to procedures presented in Chapter 3.

Standpipes of appropriate cross-sectional area are selected and

incorporated into the computer. If appropriate standpipe sizes are not available, it may be possible to insert rods (glass, brass, or stainless steel) or other solid forms into the tubes to decrease the cross-sectional area to the correct size.

The surface temperature curve for the programming device is drawn on commercial wrapping paper or stiff graph paper, cut out, and placed around the kymograph drum. The elevation of the curve in relation to the apparatus is adjusted on the kymograph. A final small adjustment is available in the platform screws which support the entire programming device. The kymograph is set to the desired speed.

Hydraulic resistors are set on the basis of reliable calibration curves, corrected for temperature if necessary. When no curves are available, the resistances must be set by trial and error according to procedures presented in Appendix B.

An initial ground temperature profile is assumed and the standpipes are filled with distilled, de-aired water to heights corresponding to the assumed temperatures. The entire apparatus is checked for air bubbles and for leaks, both of which must be eliminated.

Observations made during the analog solution are recorded on an Observation Sheet, Figure 15. The entries are obvious. The analog time is recorded by means of a stopclock and for tests of long duration a clock is used to give the total elapsed time.

The height of the water level in manometer tubes for volumetric heat standpipes is recorded every 5 to 10 minutes depending on the time scale and the desired accuracy of the results. In addition, the time when each latent heat well starts to drain and the time when it is completely drained, are recorded. These observations are used for determining the curve of frost depth versus time.

Standpipe No. 0 is a piezometer which records the surface temperature variation which can be compared with that intended to be established by the programmer.

Finally, the temperature of the water in the apparatus is observed at frequent intervals. Precautions must be taken to keep the temperature within 0.5 °C of the value for which the resistors are set.

4-03 Presenting the Computer Results. -

Observations from the analog computer solution may be plotted in a variety of ways, the most common being isotherms on a plot of depth versus prototype time.

The 32° F isotherm only may be plotted to give a curve of depth of frost penetration versus time. A direct way of determining the latter is based on observations of time when the latent heat well starts to drain and when it is completely drained. It is assumed that the lump starts to freeze at the former time and is completely frozen at the latter. Figure 16 illustrates the procedure for determining the frost depth curve.

Each horizontal line in Figure 16 represents the beginning and end of freezing for the respective lump. If a latent heat well is not completely drained before the adjacent one begins, half of the overlap time is included in the time for drainage of both wells. If different amounts of latent heat are represented by the two wells, the overlap time is divided in proportion to the volume of the wells based on the assumption that freezing in this region is proportional to time. This overlap, which is theoretically not possible, occurs because the latent heat will discharge its volume under a slightly varying head.

When a well is partly drained, the depth of freezing in the soil lump is assumed to be proportional to the amount of water that is drained from the well.

4-04 Verification of Computer Results. -

Operation of analog computers of any type are subject to errors which may be contributed solely by the operators, such as errors in programming or which may be caused by mechanical difficulties in the computer, such as leakage and blocking of resistors by air bubbles or impurities. Also, an analog solution obtained by "lumping" time or space variables involves some degree of inaccuracy, which is inherent with the finite difference approximation.

One may achieve a verification of computer results by reprogramming the problem to different scales. If the results turn out to be in agreement with the first set, there is good indication that the results are correct. This procedure is particularly favorable in checking errors in

scaling and setting. However, it involves time and labor. Probably satisfactory results can be obtained by simply double checking all initial scalings and settings before and after the problem is run.

The foremost method for checking errors caused by mechanical difficulties is by constant calibration of all contributing components of the computer. Any mechanical difficulty will be noted immediately if deviation is observed in the calibration chart.

Indeed, analog results can always be verified by comparison with results of analytical or numerical solutions. The drawback is that analytical solutions exist for only a limited number of heat flow problems, and thus for complicated soil profiles and boundary conditions, numerical solutions will involve tedious labor. Indeed, it is because of such drawback that the hydraulic analog computer is designed.

The analog computer inherits an inaccuracy in representing soil in lumps. The degree of such inaccuracy depends among other things on the size of each lump and the magnitude of thermal gradients. A study of the magnitude of this inaccuracy was carried out by Scott (21) by comparing analog results of different lump sizes with results of an analytic solution, and it was found that the error involved is negligible when the solutions are programmed according to the procedure recommended in Chapter 4.

In summary, the hydraulic analog computer, the design and operation of which is presented in this report, has been found to be a very useful tool for solving complex one-dimensional freezing and thawing problems in soil. Indeed, its applicability for solving other diffusion problems in engineering is practically limitless. Thus, to use the computer to solve consolidation problems in soil, for example, one needs knowledge only of the analogous physical relationships. The programming steps are similar.

BIBLIOGRAPHY

1. Aldrich, H. P. and Paynter, H. M.,"Analytical Studies of Freezing and Thawing of Soils". First Interim Report to N.E.D. Corps of Engineers, U. S. Army, June 1953.

2. Aldrich, H. P.,"Frost Penetration Below Highway and Airfield Pavements", Paper presented at Annual Meeting, Highway Research Board, Washington D.C., 1956.

3. Moore, A. D.,"The Hydrocal" Ind. and Eng. Chem. v. 28, p. 704, 1936.

4. Lukyanov, V. S., "Hydraulic Apparatus for Engineering Computations," Izvestiya Academiyi Nauk U.S.S.R., Otdeleniye Technicheskikh Nauk, No. 2., pp. 53 to 67, 1939. (Translation by Dimitry Vergun, M.I.T.) Arctic Construction and Frost Effects Laboratories, Boston, Mass., June 1955.

5. Lamb, J. F., "Sun Effect and Heat Flow Through Brick Walls as Studied With the Hydrocal", Heating and Ventilating, Vol. 33, No. 8, pp. 32-35, 1936.

6. Backstrom M., "Avkylningsforlopp och temperatur vandringar vid complicerade kylproblem, belysta med hydraulisk raknemaskin", Tidsskr., Varme, Ventillation, Sanitet, och Kyltekniik, Vol. 19, p. 113, 1948.

7. Backstrom, M., "Kaltetechnik", Verlag G. Braum, Karksruhe, second edition, 1953.

8. Leopold, C. S.,"Hydraulic Analogue for the Solution of Problems of Thermal Storage, Radiation, Convection and Conduction". Trans. A.S.H.V.E. p. 389, 1948.

9. Leopold, C. S.,"Design Factors in Panel and Air Cooling Systems", Heating, Piping, and Air Conditioning, pp 115-128, May 1951.

10. Barron, R. A., "Viscous Flow Tube Model", Proc. 2nd Int. Conf. Soil Mech. and Found. Eng. V. III, Paper IIg 13, 1948.

11. Coyle, M. B., "An Airflow Analogy for the Solution of Transient Heat Conduction Problems", British Journal Applied Physics, Vol. 2, pp 12-17, 1951.

12. Aldrich, H. P., "Analysis of Foundation Stresses and Settlements at Hayden Library," Mass. Inst. of Technology, Sc.D. Thesis, 1951, unpublished.

13. Rigas, A. C., "The Consolidation Analogy Model," Mass. Inst. of Technology, S.B. Thesis, 1951, unpublished.

BIBLIOGRAPHY (Contd.)

14. Juhasz, I. S., and Hooper, F. C., "Hydraulic Analogue for Studying Steady State Heat Exchange", Ind. and Eng. Chem. Vol. 45 p. 1359, 1953.

15. de Laclemandiere, "Etude experimentale de la transmission de la chaleur en regime variable". Chaleur et Industrie Vol. 28, No. 269, pp 293-308, 1947. Chaleur et Industrie Vol. 29, No. 270, pp 14-28, 1948.

16. Ramachandran, A., Electrotechnics No. 23, pp 110-115, 1951.

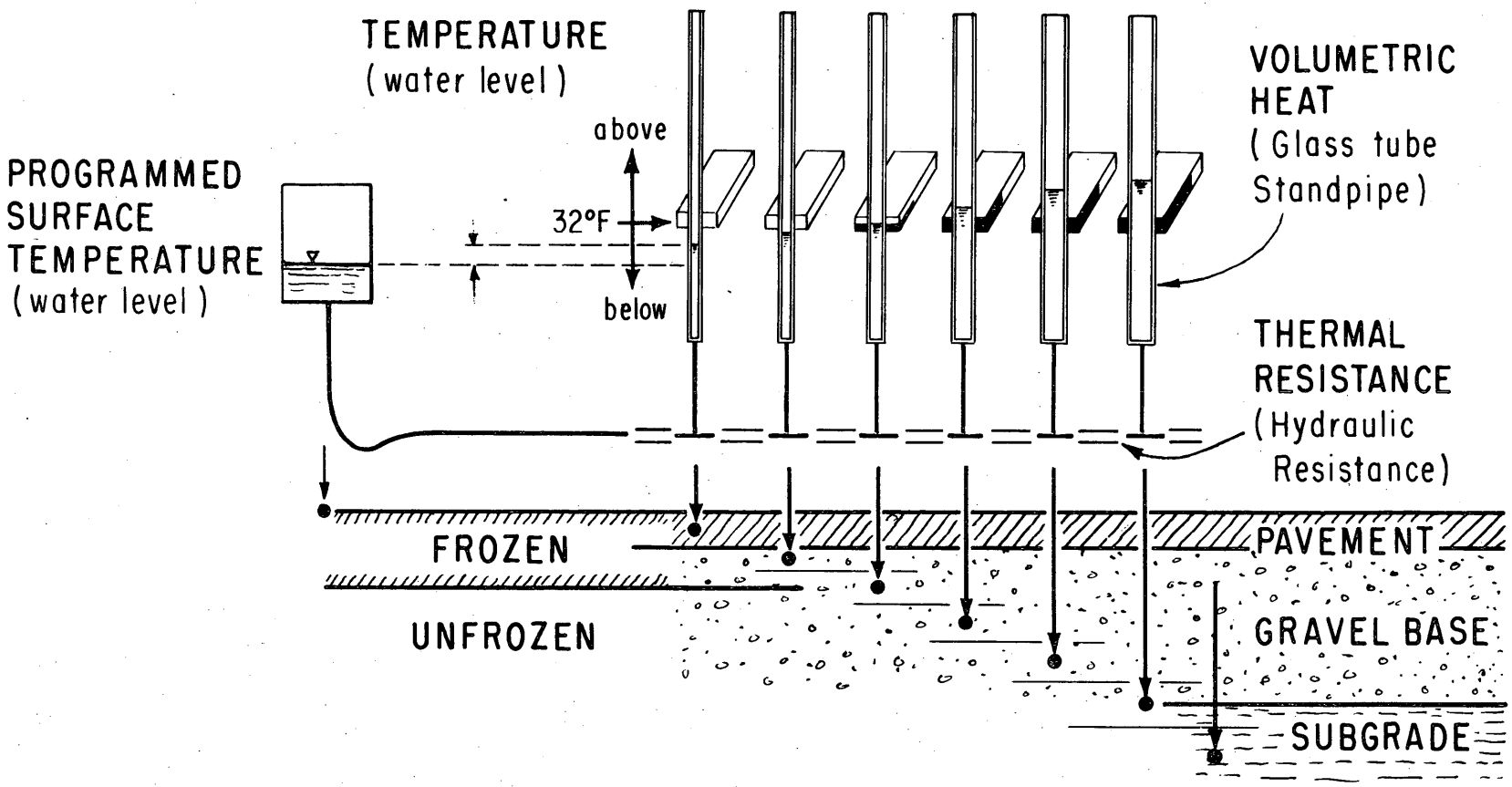
17. Lambe, T. W., "Soil Testing for Engineers", John Wiley, New York, 1951.

18. Taylor, D. W., "Fundamentals of Soil Mechanics", John Wiley, New York, 1948.

19. Rouse, H., "Fluid Mechanics for Hydraulic Engineers", McGraw-Hill, New York, 1938.

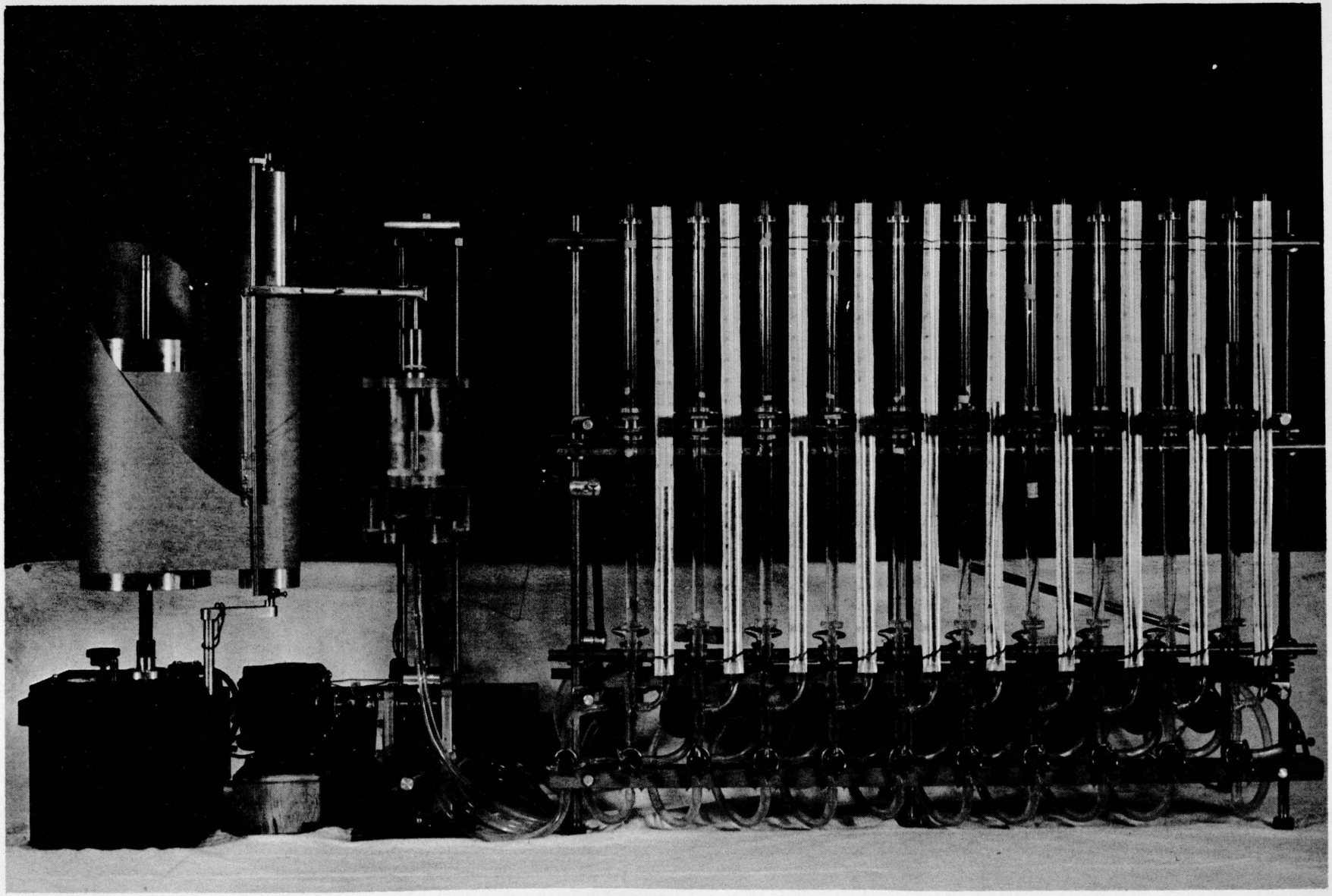
20. Nordal, R., "An Investigation of Frost Penetration in Soil by Hydraulic Analog Computer", Mass. Institute of Technology, S.M. Thesis, 1955, unpublished.

21. Scott, R. F., "Investigation of Factors Affecting Heat Transfer at the Air-Ground Interface with Special Reference to Freezing and Thawing Problems under Airfield Pavements", Mass. Institute of Technology, Sc.D. Thesis, 1955, unpublished.



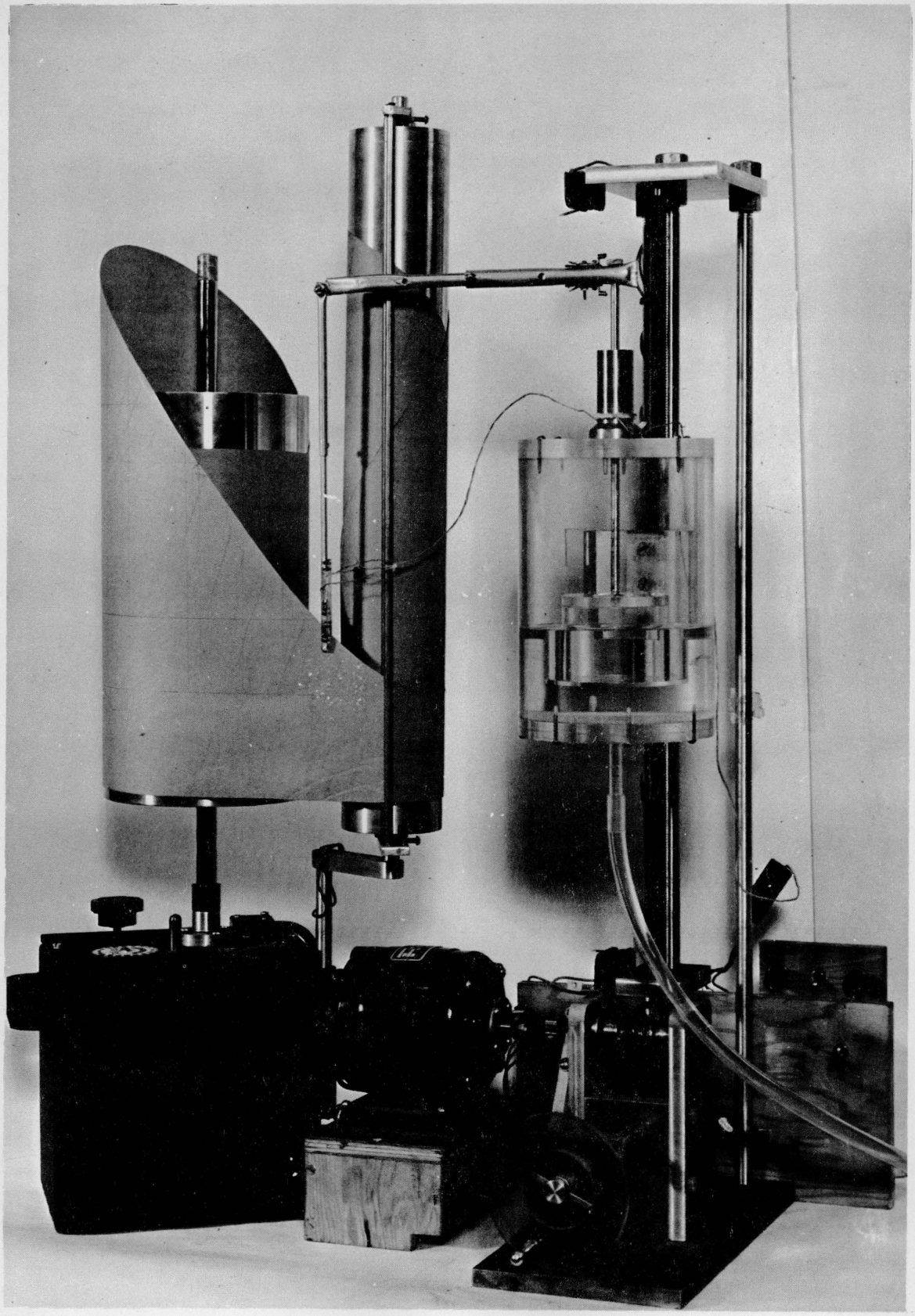
HYDRAULIC ANALOG TO FROST PENETRATION

FIGURE 1



HYDRAULIC ANALOG COMPUTER

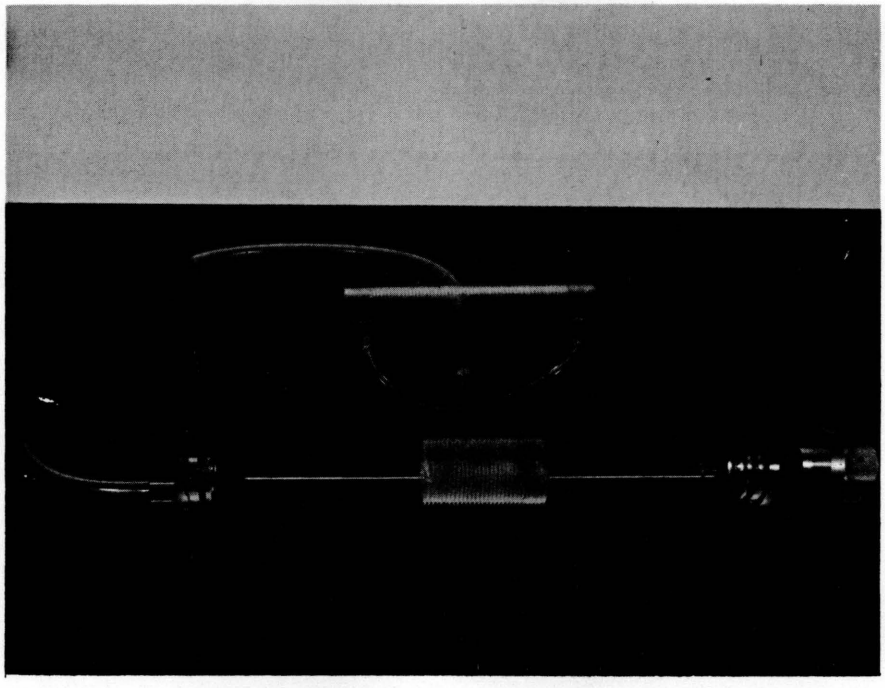
FIGURE 2



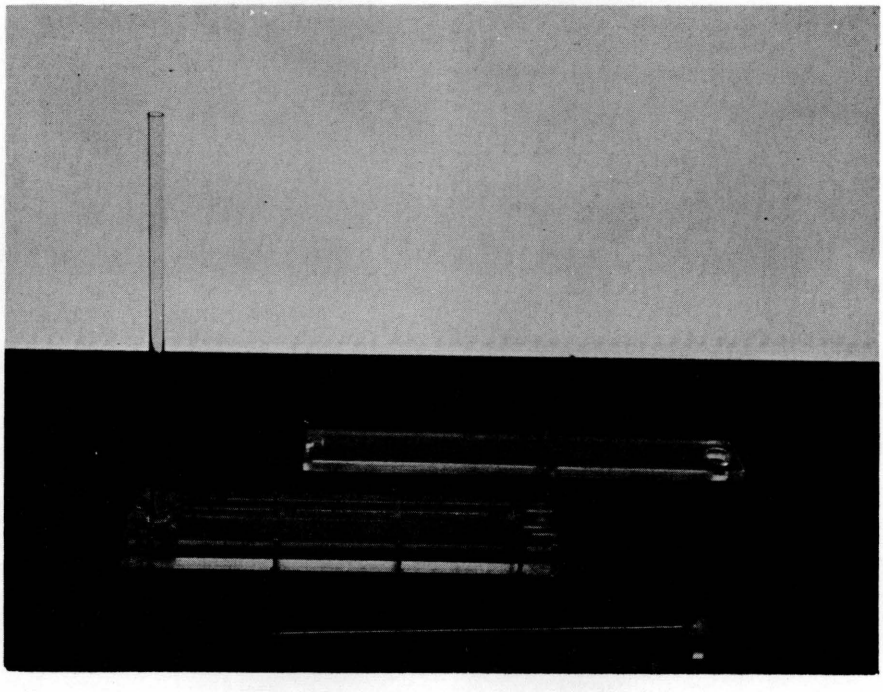
PROGRAMMER

FIGURE 3

COMPUTER COMPONENTS REAR VIEW FIGURE 4



RESISTOR TYPE A

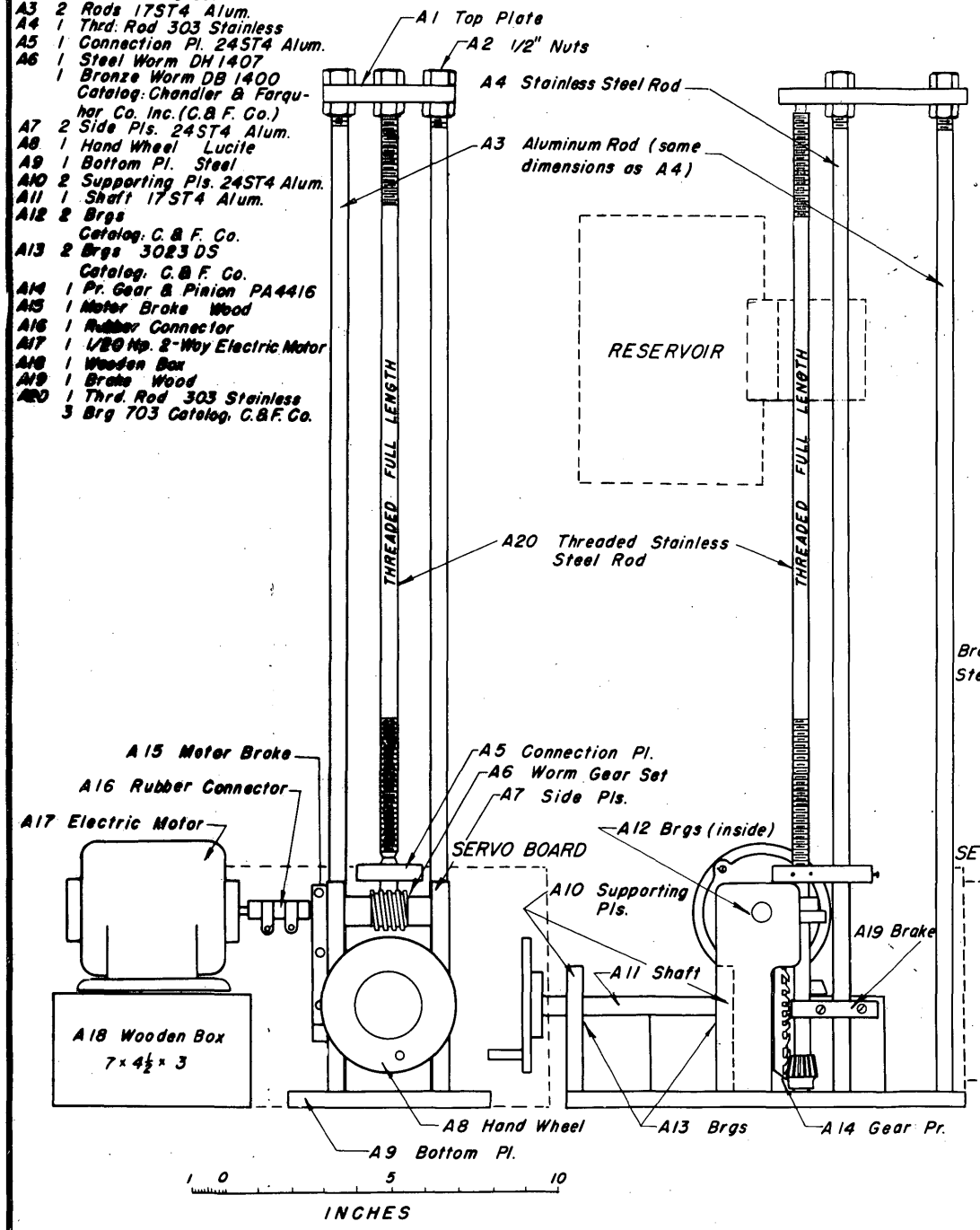


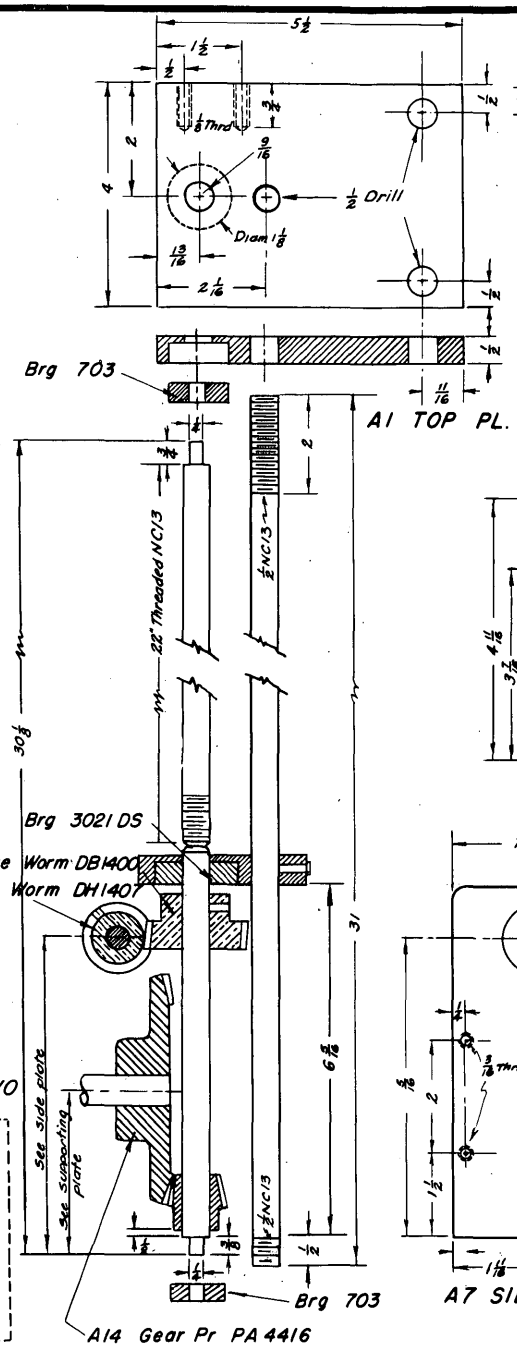
LATENT HEAT WELL

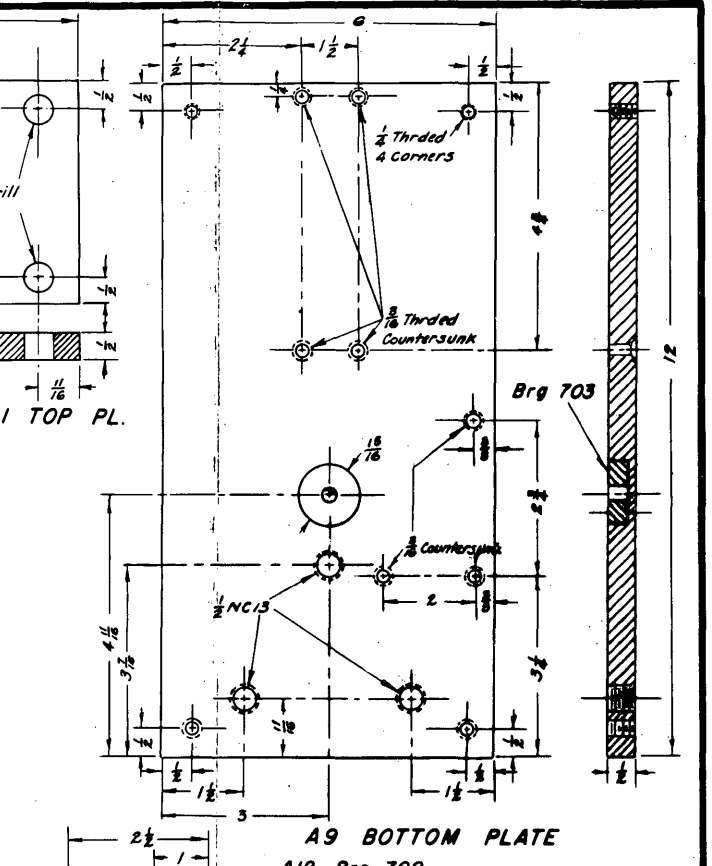
FIGURE 5

BILL OF MATERIALS

A1	1	Top Pl. 24ST4 Alum.
A2	6	1/2" Nuts Steel
A3	2	Rods 17ST4 Alum.
A4	1	Thrd. Rod 303 Stainless
A5	1	Connection Pl. 24ST4 Alum.
A6	1	Steel Worm DH 1407
	1	Bronze Worm DB 1400 Catalog: Chandler & Farquhar Co. Inc. (C.& F. Co.)
A7	2	Side Pls. 24ST4 Alum.
A8	1	Hand Wheel Lucite
A9	1	Bottom Pl. Steel
A10	2	Supporting Pls. 24ST4 Alum.
A11	1	Shaft 17ST4 Alum.
A12	2	Brgs Catalog: C. & F. Co.
A13	2	Brgs 3023 DS Catalog: C. & F. Co.
A14	1	Pr. Gear & Pinion PA4416
A15	1	Motor Brake Wood
A16	1	Rubber Connector
A17	1	1/20 Hp. 2-Way Electric Motor
A18	1	Wooden Box
A19	1	Brake Wood
A20	1	Thrd. Rod 303 Stainless
	3	Brg 703 Catalog: C.& F. Co.







ALL DIMENSIONS IN INCHES

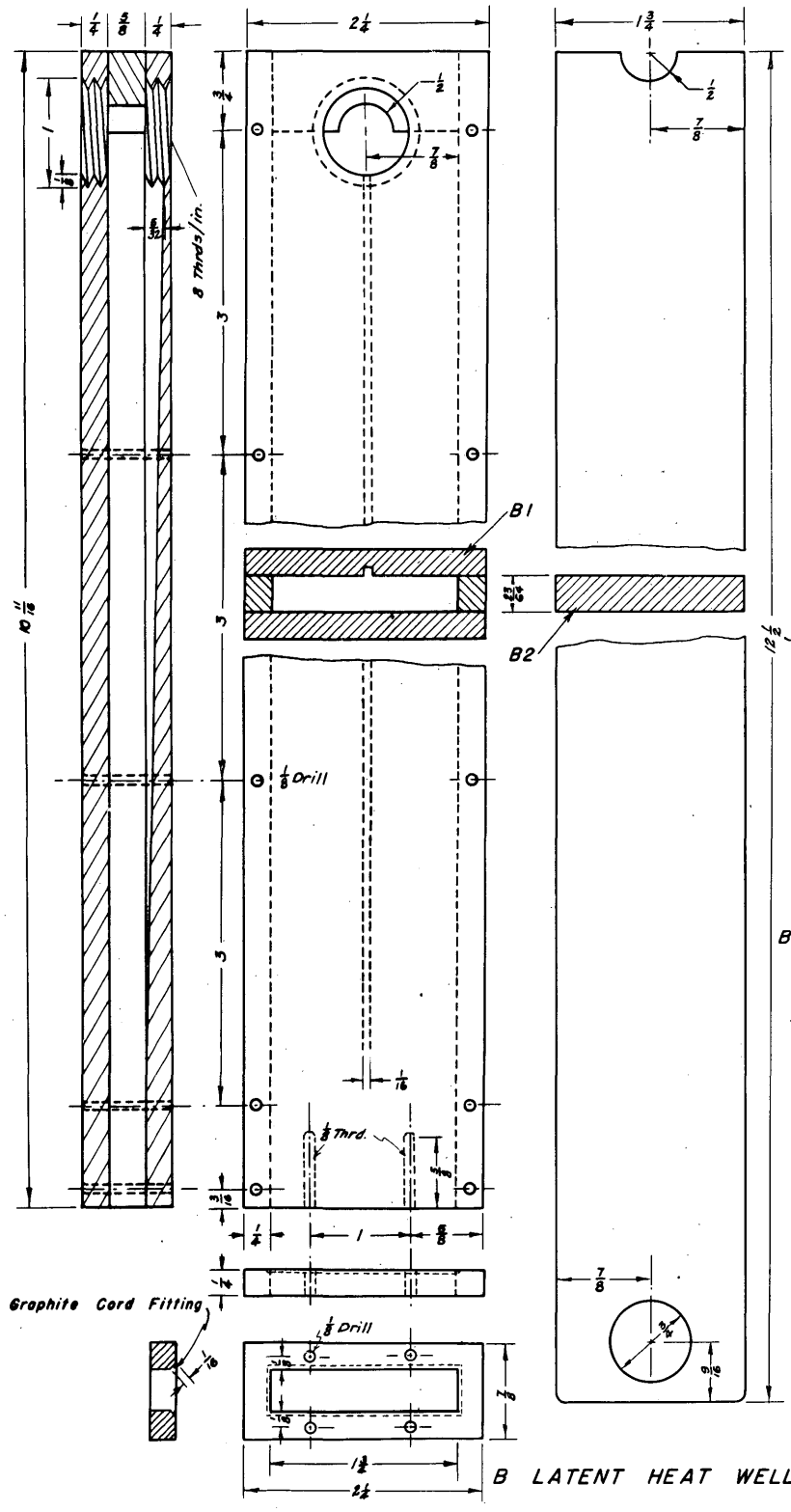
ARCTIC CONSTRUCTION AND FROST EFFECTS LAB.
NEW ENGLAND DIV., CORPS OF ENG., U. S. ARMY

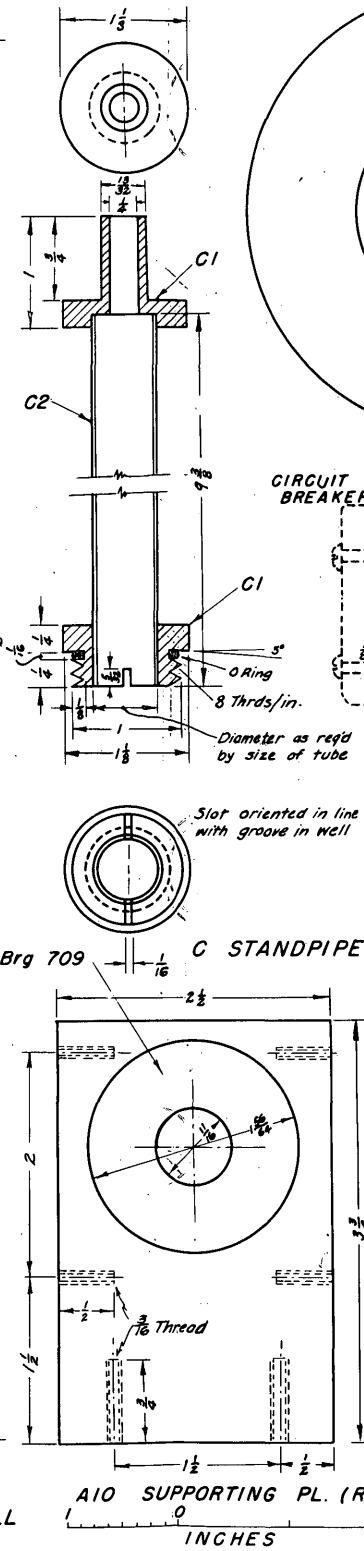
PROGRAMMER FRAME

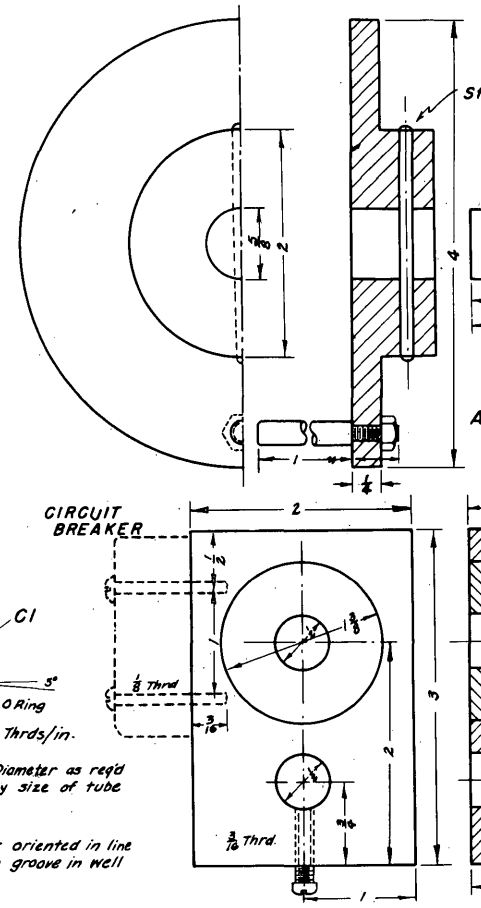
HYDRAULIC ANALOG COMPUTER
CONTRACT DA-19-016-ENG-2743

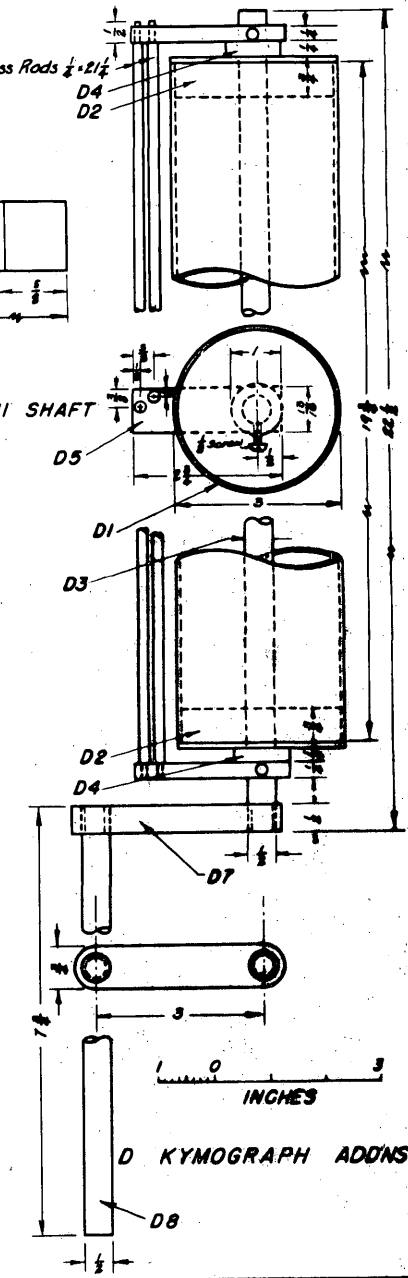
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASS.
DRAWN BY C.Y.L. L.T.R.
SCALE AS NOTED
DATE APRIL 1956

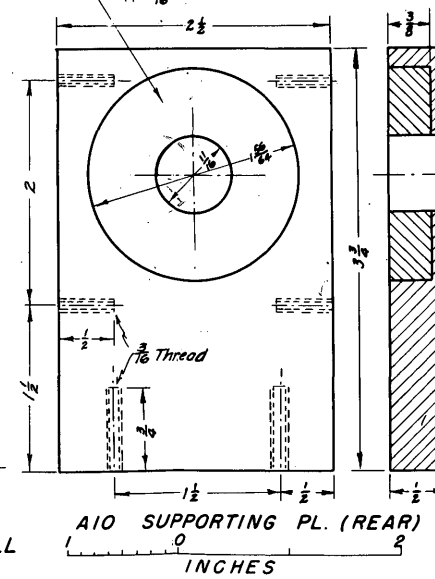
FIGURE 6











BILL OF MATERIALS

A5	1	Connection Pl. 24ST4 Alum.
	1	Brg 3021 DS
A8	1	Hand Wheel Lucite
	1	1/8" Steel Pin
	1	Handle Brass
A10	1	Supporting Pl. 24ST4 Alum.
	1	Brg 709
A11	1	Shaft 24ST4 Alum.
B1	1	Box Lucite
B2	1	Piston Lucite
	14	1/8" Nuts & Bolts
C1	1	Cap & Socket Lucite
C2	1	Tube Glass or Lucite
C3	1	O Ring
D1	1	Cylinder Wall 1/16" Thick Alum.
D2	2	Caps 24ST4 Alum.
D3	1	Shaft " "
D4	2	Rings " "
D5	2	Bars " "
D6	2	Rods Brass
D7	1	Bar 24ST4 Alum.
D8	1	Rod " " "

(Brg Details See Catalog Chandler & Farquhar Co.)
(ALL DIMENSIONS IN INCHES)

ARCTIC CONSTRUCTION AND FROST EFFECTS LAB
NEW ENGLAND DIV., CORPS OF ENGI U. S. ARMY

LATENT HEAT WELL & KYMOGRAPH ADDITIONS

HYDRAULIC ANALOG COMPUTER
CONTRACT DA-19-016-ENG-2743

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASS.
DRAWN BY C.Y.L.
SCALE AS NOTED
DATE APRIL 1956

FIGURE 7

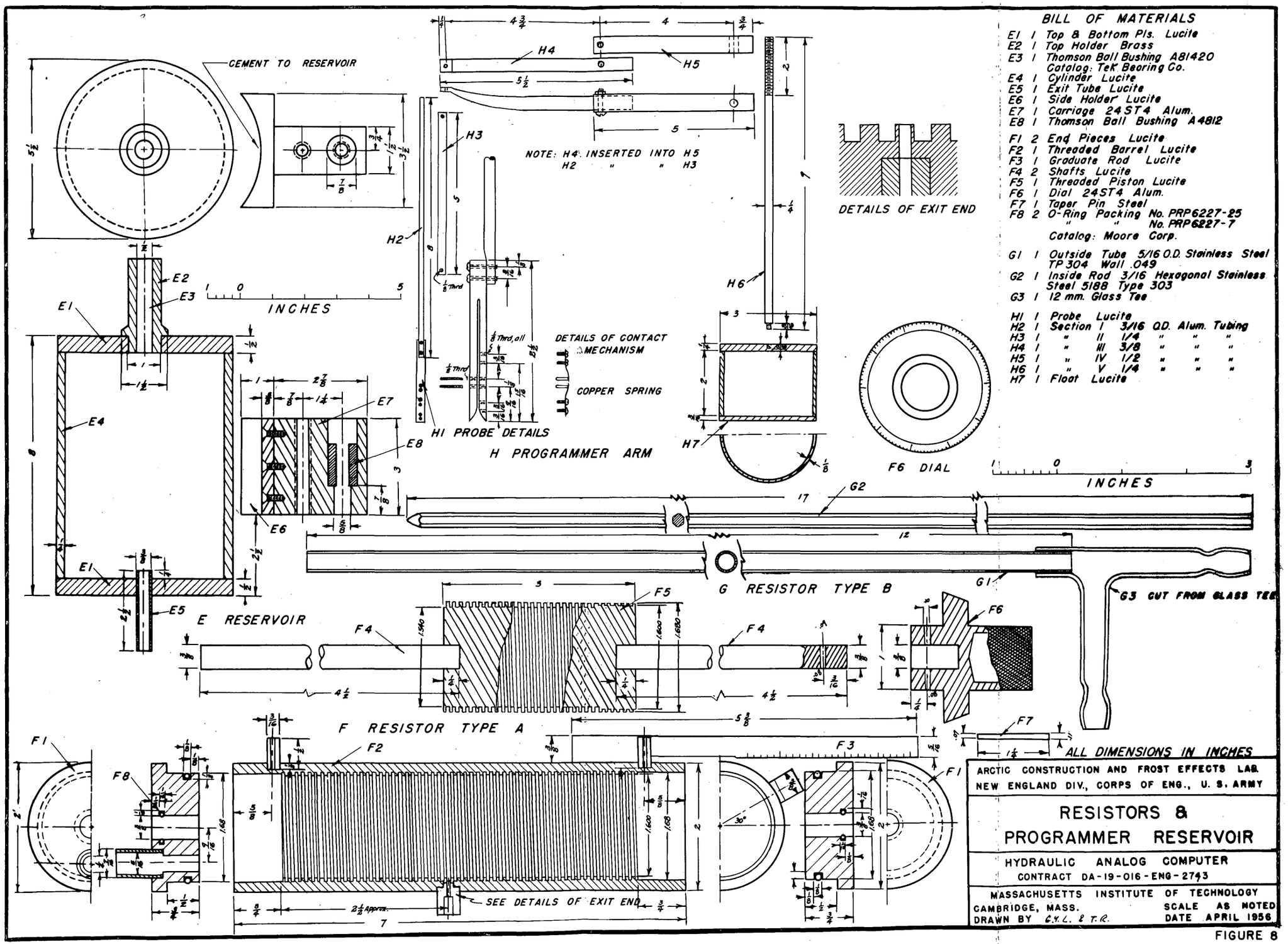
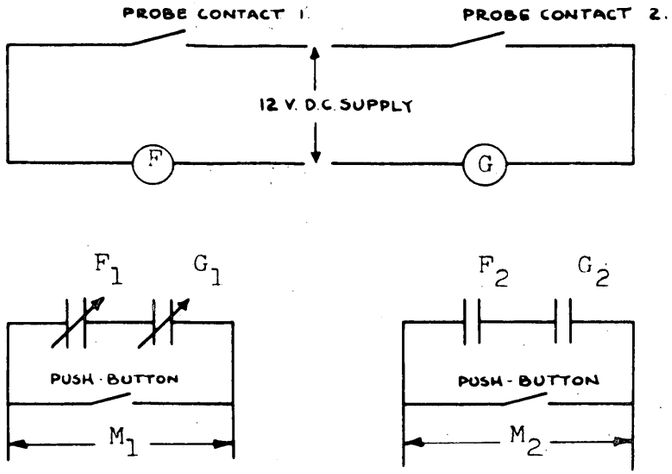
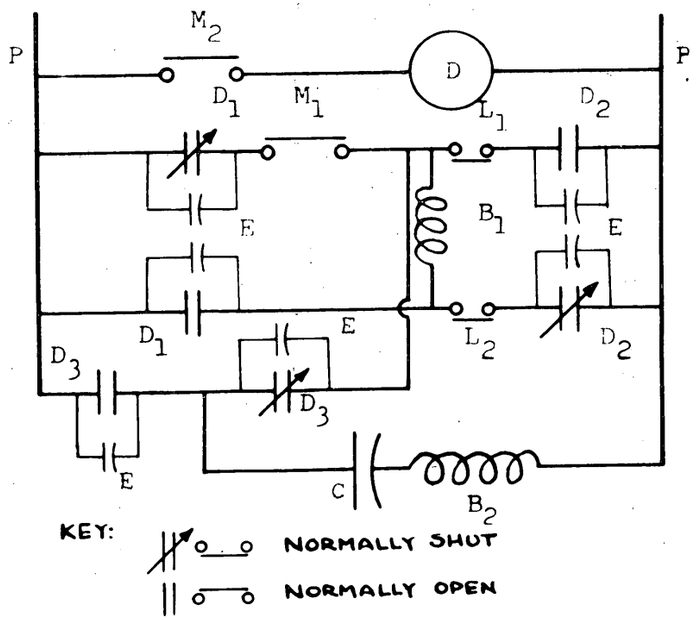

BILL OF MATERIALS
E1 1 Top & Bottom Pls. Lucite
E2 1 Top Holder Brass
E3 1 Thomson Ball Bushing A81420
Catalog: TeK Bearing Co.
E4 1 Cylinder Lucite
E5 1 Exit Tube Lucite
E6 1 Side Holder Lucite
E7 1 Carriage 24ST4 Alum.
E8 1 Thomson Ball Bushing A4812
F1 2 End Pieces Lucite
F2 1 Threaded Barrel Lucite
F3 1 Graduate Rod Lucite
F4 2 Shafts Lucite
F5 1 Threaded Piston Lucite
F6 1 Dial 24ST4 Alum.
F7 1 Taper Pin Steel
F8 2 O-Ring Packing No. PRP6227-25
" " No. PRP6227-7
Catalog: Moore Corp.
G1 1 Outside Tube 5/16 O.D. Stainless Steel TP 304 Wall .049
G2 1 Inside Rod 3/16 Hexagonal Stainless Steel 5188 Type 303
G3 1 12 mm. Glass Tee
H1 1 Probe Lucite
H2 1 Section I 3/16 O.D. Alum. Tubing
H3 1 " II 1/4 " " "
H4 1 " III 3/8 " " "
H5 1 " IV 1/2 " " "
H6 1 " V 1/4 " " "
H7 1 Float Lucite
CEMENT TO RESERVOIR
NOTE: H4 INSERTED INTO H5
H2 " " H3
DETAILS OF EXIT END
DETAILS OF CONTACT MECHANISM
COPPER SPRING
H1 PROBE DETAILS
H PROGRAMMER ARM
F6 DIAL
INCHES
E RESERVOIR
G RESISTOR TYPE B
G3 CUT FROM GLASS TEE
F RESISTOR TYPE A
SEE DETAILS OF EXIT END
ALL DIMENSIONS IN INCHES
ARCTIC CONSTRUCTION AND FROST EFFECTS LAB.
NEW ENGLAND DIV., CORPS OF ENG., U. S. ARMY
RESISTORS & PROGRAMMER RESERVOIR
HYDRAULIC ANALOG COMPUTER
CONTRACT DA-19-016-ENG-2743
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASS.
DRAWN BY C.Y.L. & T.R.
SCALE AS NOTED
DATE APRIL 1956

FIGURE 8



a. PROBE AND RELAY CIRCUITS



b. MOTOR CIRCUIT

Electrical Elements Key

Symbol	Element
B_1, B_2	Field coils of 1/20 h.p. A.C. split-phase capacitor start and run motor.
C	5 mf. motor capacitor.
D	3PDT motor relay coil.
D_1, D_2, D_3	Contacts of motor relay.
E	0.002 mf. capacitors across motor relay contacts to inhibit sparking.
F	First probe relay coil.
F_1, F_2	Contacts of first probe relay.
G	Second probe relay coil.
G_1, G_2	Contacts of second probe relay.
L_1, L_2	Limit switches to prevent reservoir over-running.
M_1, M_2	Up and down switches for motor, operated by probe relays.
P	115V. A.C. power supply.

A 5A. fuse is also inserted in the motor circuit.

Two push-button switches are used to control the starting position of the reservoir.

PROGRAMMER CIRCUIT DIAGRAM

FIGURE 9

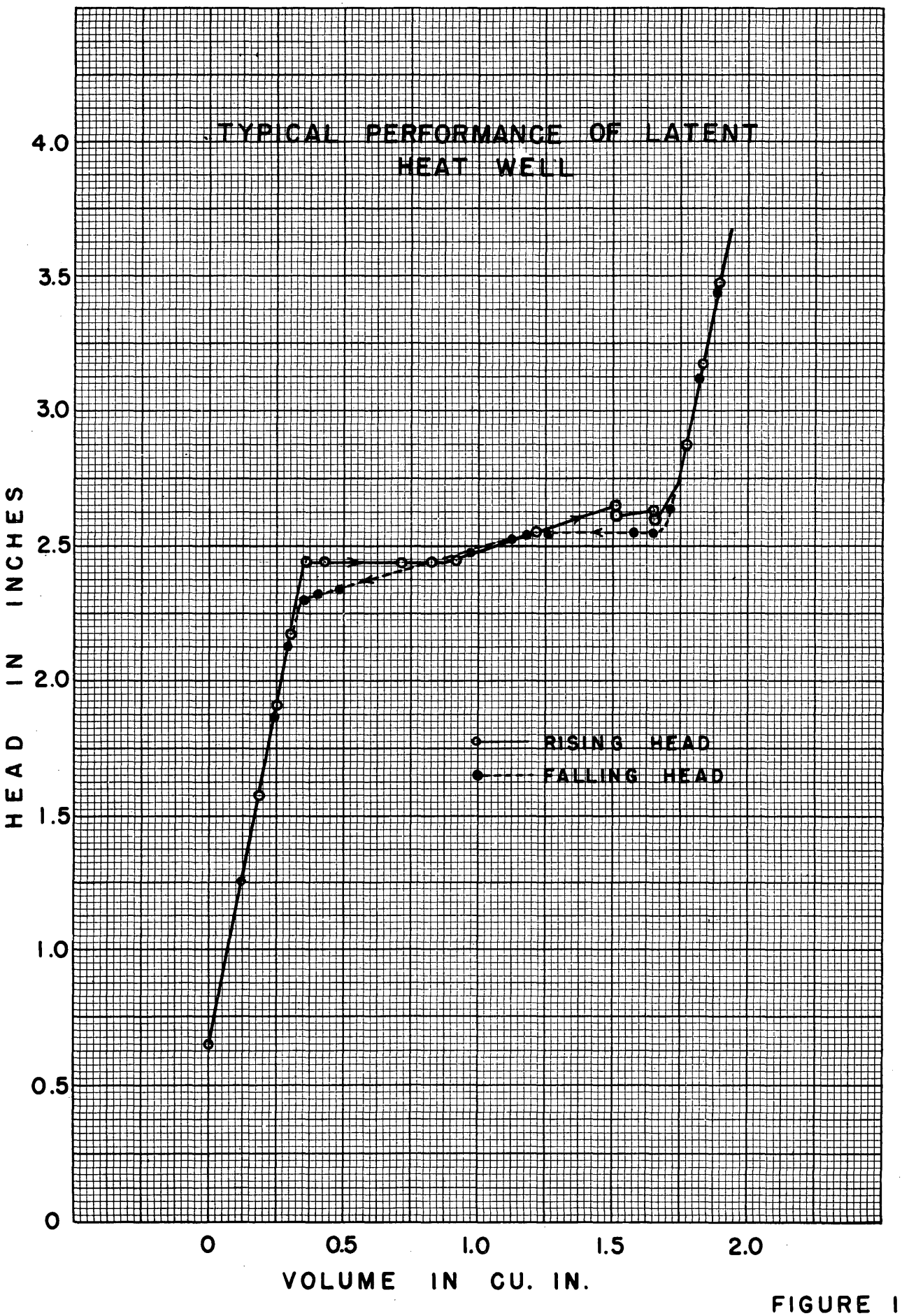

TYPICAL PERFORMANCE OF LATENT HEAT WELL
HEAD IN INCHES
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.5
1.0
1.5
2.0
VOLUME IN CU. IN.
RISING HEAD
FALLING HEAD

FIGURE 10

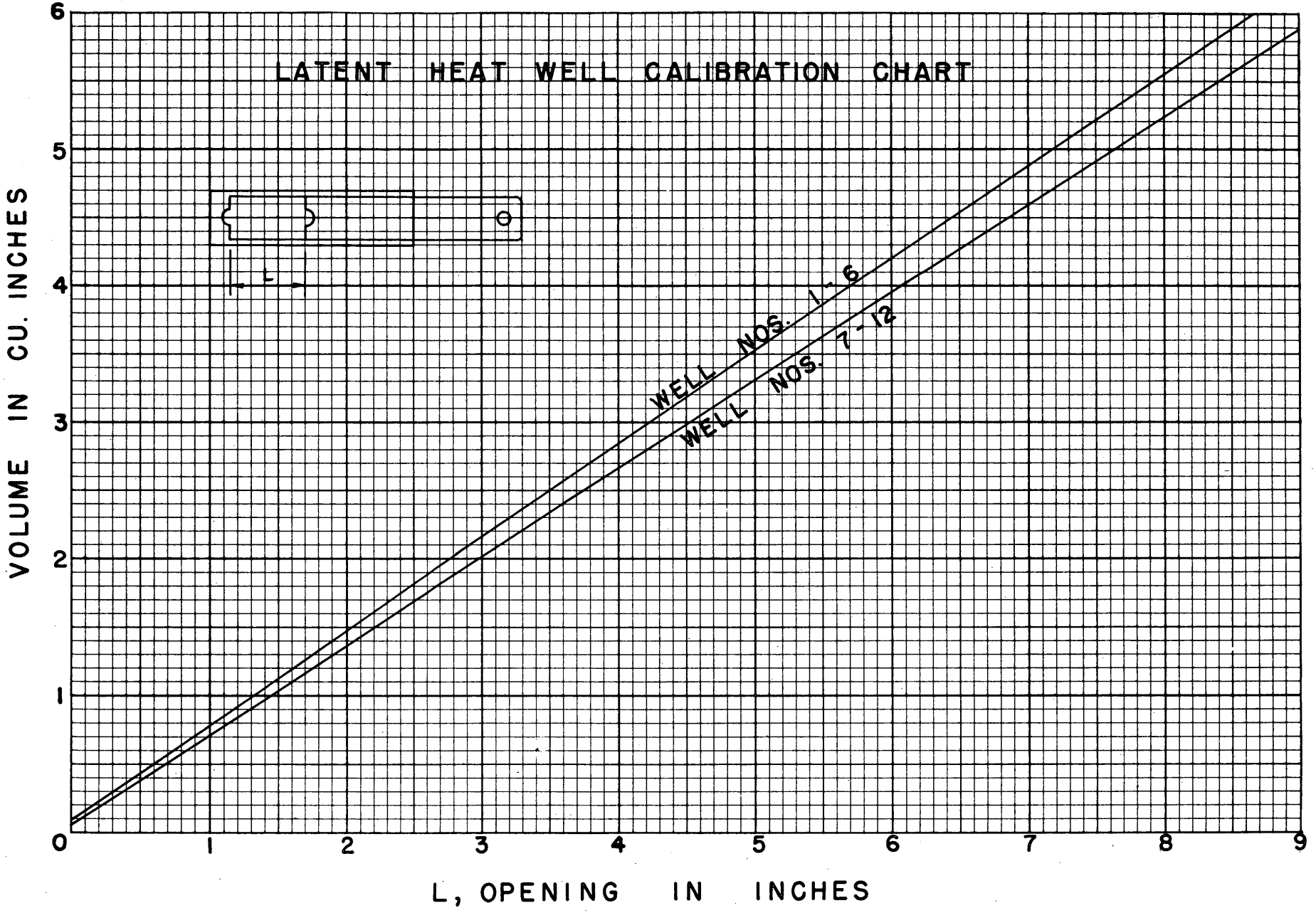

LATENT HEAT WELL CALIBRATION CHART
VOLUME IN CU. INCHES
L, OPENING IN INCHES
WELL NOS. 1-6
WELL NOS. 7-12
L
0
1
2
3
4
5
6
7
8
9

FIGURE 11

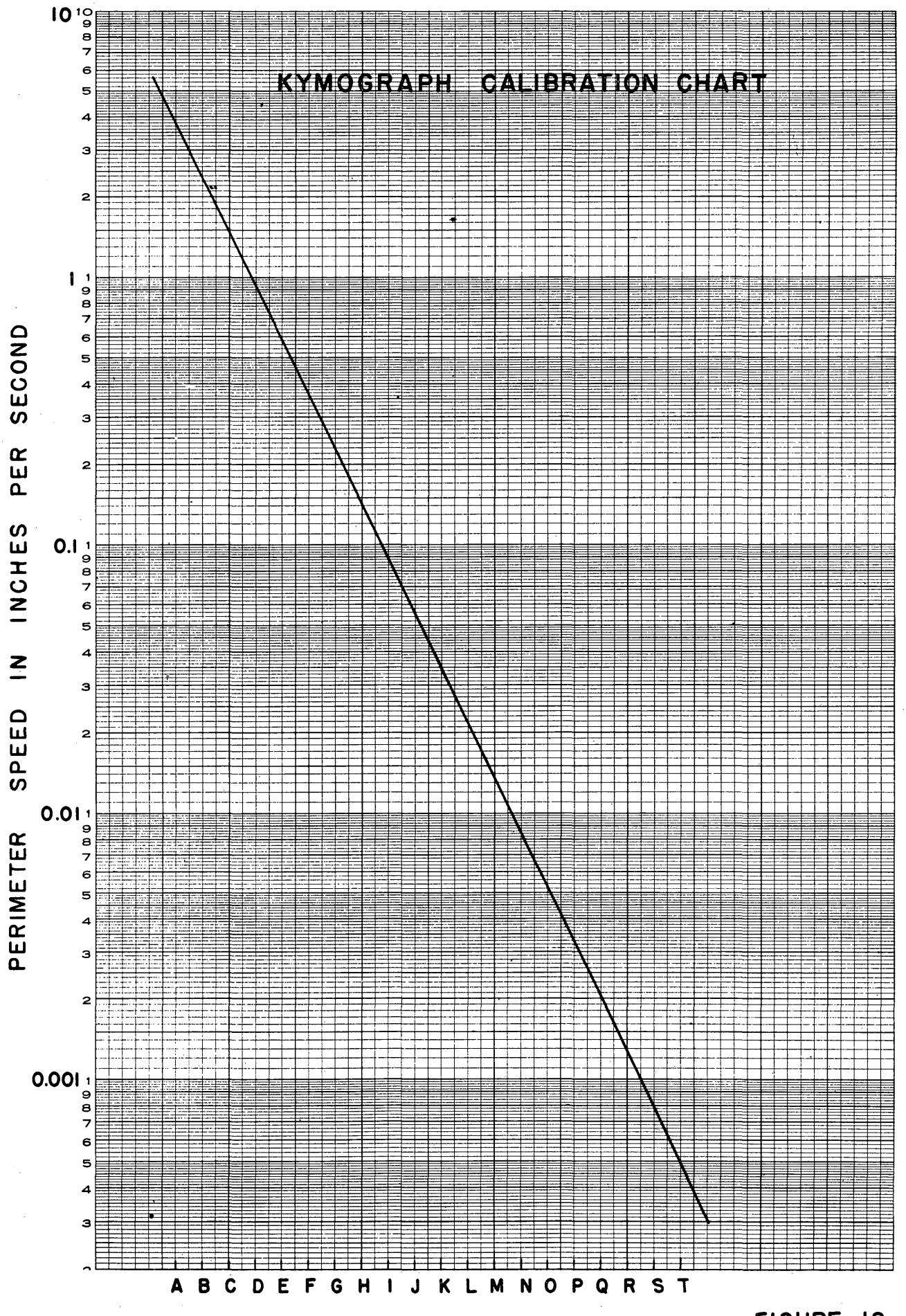
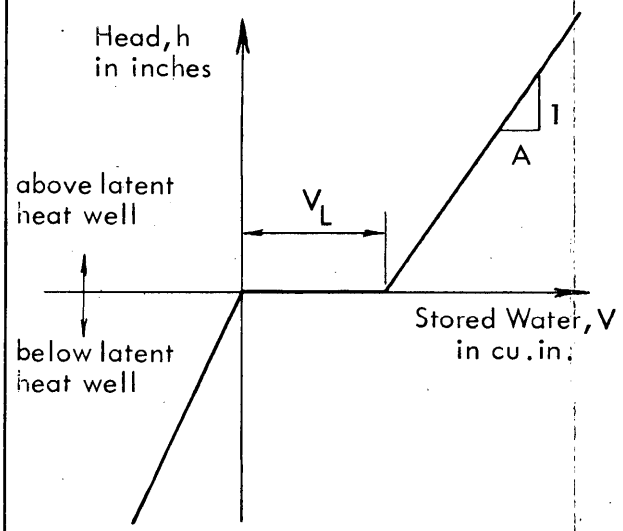

KYMOGRAPH CALIBRATION CHART
PERIMETER SPEED IN INCHES PER SECOND
10
1
0.1
0.01
0.001
A B C D E F G H I J K L M N O P Q R S T
KYMOGRAPH DIAL READING

FIGURE 12

a. IDEAL STORED WATER VS HEAD DIAGRAM FOR AN ELEMENT OF THE COMPUTER REPRESENTING SOIL "LUMP".



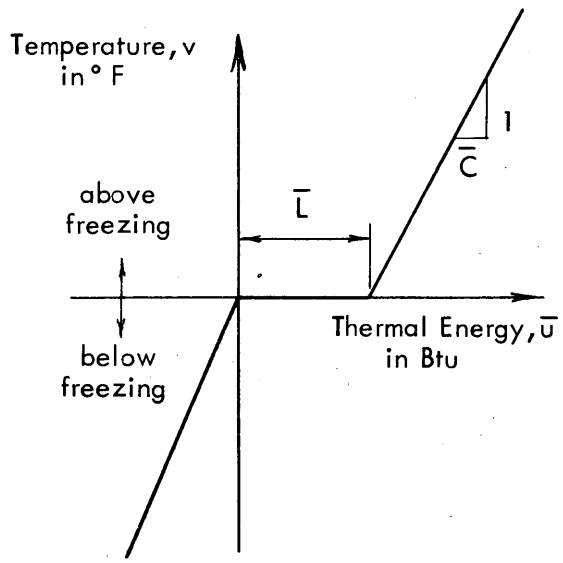
A, Standpipe and manometer area, sq.in.

V_L, Latent heat well vol., cu.in.

R_a, Analog resistance, min.per sq.in.

t_a, Analog time, min.

b. IDEAL THERMAL ENERGY VS TEMPERATURE DIAGRAM FOR A SOIL "LUMP" OF UNIT AREA (SQ.FT.) AND THICKNESS Th.



$\bar{C}$, Volumetric heat, Btu per° F

$\bar{L}$, Latent heat, Btu

R_a, Thermal resistance, °F hr. per Btu

t_p, Time in nature, hr.

SCALING RELATIONSHIPS

ANALOG — TO — PROTOTYPE

Temperature Scale, $V_s = \frac{h}{v}$; in. per °F

Thermal Energy Scale, $U_s = \frac{A}{\bar{C}} \cdot V_s$; cu. in. per Btu

Time Scale, $T_s = \frac{t_a}{t_p} = \frac{R_a A}{R_p \bar{C}}$; min. per hr.

FIGURE 13

HYDRAULIC ANALOGUE COMPUTER
SCALING SHEET

TEST NO.: 9

PROBLEM: Frost Penetration below Asphalt Concrete Pavement for Step Change in Surface Temperature

DATE: 16 July, 1955

COMPUTED BY: RN

TIME SCALE 1 MIN = 2 DAYS; 1°F = 0.2 INCH; 1 BTU = .00195 INCH³;

SOIL PROFILE	L BTU/FT³	C BTU/°F FT³	k BTU/°F HR FT	SLIDE OPENING INCH	LATENT HEAT INCH³	LATENT HEAT BTU	STAND-PIPE AREA (INCH²)	ANALOGUE PROFILE (INCHES)	ANALOGUE PROFILE	1" = (INCHES)	R_p °F HR/BTU	R_a MIN/IN²	Setting (INCHES)
	2	3	4	5	6	7	8		10		12	13	14
3" A.C.	0	28	0.8	0		0	.068	3	1	1.5	.156	0.34	16.18
6" Base				0.44	0.415	213	.056	3	2	3		0.61	15.67
γ_d = 110 pcf.	850	23	1.0								.25	0.54	15.78
ω = 5%				0.44	0.415	213	.056	3	3	2.68		0.64	15.80
21.5" Subbase				1.38	1.045	537	.109	5.37	4	5.37	.344	0.74	15.76
γ_d = 120 pcf	1200	25	1.30	1.33	"	"	.109	5.37	5	5.37	.344	0.74	15.41
ω = 7%				1.33	"	"	.109	5.37	6	5.37	.344	0.74	15.46
				1.58	"	"	.109	5.37	7	7.82		0.91	15.91
Subgrade				4.31	2.83	1450	.225	10.26	8	10.26	.50	1.08	15.40
γ_d > 120 pcf	1700	27	1.70	4.31	"	"	.225	10.26	9	10.26	.50	1.08	15.92
ω = 10%				4.31	"	"	0225	10.26	10	11.23	.55	1.18	(TURNS) 0.35
				5.14	3.37	1730	.267	12.2	11	12.2	.60	1.28	0.37
				5.14	"	"	.267	12.2	12	15.55	.76	1.63	0.60
							.415	18.9	13	18.9	.925	1.98	0.50
							.415	18.9	14	34.55	1.70	3.63	1.50
							1.10	50.2	15	53.8	2.63	5.60	4.55
							1.26	57.4	16	80.2	3.94	8.45	2.50
							2.26	103	17	150	7.35	15.70	6.00
							4.32	197	18	283	13.90	29.70	13.00
							8.1	369	19	500	24.4	52.2	22.00
							13.8	630	20				

RESISTOR B

RESISTOR A

FIGURE 14

HYDRAULIC ANALOGUE COMPUTER OBSERVATION SHEET

OBSERVED BY: RN TEST NO. 9

DATE: 16 July 1955 SHEET 1 OF 1

PROBLEM: Frost Penetration below Asphalt Concrete Pavement for Step Change in Surface Temperature

DATE AND HOUR	ELAPSED TIME 1 MIN= 2 DAYS ANALOGUE (MIN)	PROTOTYPE (DAYS)	STANDPIPES: TEMPERATURE SCALE 0.2 INCH = 1° F 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	REMARKS
				0.13	0.38	0.63	0.97	1.42	1.86	2.31	2.96	3.82	4.67	5.61	6.63	7.92	9.50	12.46	16.94	23.62	36.12	59.72	101.42	Depth in ft. below Surface
12:43	0	0	25	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	41.8	28°C temp. water
					0:20	0:50	1:40	4:00	7:20	11:20	16:10	28:50	41:30	59:30	83:15	Time	latent heat well begins to empty							
					0:40	1:30	3:50	7:20	11:40	16:10	29:40	48:45	71:-	100:-	118:-	Time	"	"	"	completely emptied				
	5	10	23	24.-	26	27.6	29.8	32-	32.9	33.8	34.7	36	37	37.9	38.8	39.7	40.6	41.5	41.8	41.8	41.8	41.8	41.8	28°C
	10	20	23	23.3	25.2	26.3	27.9	29.4	31.3	32-	32.8	33.9	34.9	35.5	36.4	37.5	38.5	40.3	41.3	41.7	41.8	41.8	41.8	
	16	32	23	23.8	24.7	25.6	26.9	28.1	29.4	30.7	32-	32.8	33.7	34.3	35.1	36.2	37.2	39.3	40.7	41.5	41.8	41.8	41.8	28°C
	20	40	23	23.6	24.6	25.4	26.7	28	29.3	30.6	32-	32.7	33.4	34.1	34.7	35.7	36.7	38.6	40.3	41.2	41.8	41.8	41.8	
	25	50	22.9	23.5	24.5	25.4	26.6	27.7	28.9	30.1	31.4	32.2	32.8	33.5	34.1	35.1	36.1	38	39.7	40.8	41.7	41.8	41.8	
	30	60	22.9	23.4	24.4	25.1	26.3	27.3	28.4	29.5	30.6	32	32.6	33.2	33.8	34.7	35.6	37.5	39.3	40.6	41.7	41.8	41.8	28°C
	35	70	22.9	23.4	24.2	25	25.9	27	28.1	29.2	30.4	32	32.5	33-	33.6	34.5	35.2	37	38.3	40.2	41.6	41.7	41.8	
	40	80	23	23.4	24.3	24.9	26.7	26.9	28	29	30	31.5	32.2	32.7	33.2	34.2	34.9	36.6	38.5	39.9	41.5	41.7	41.8	
	45	90	23	23.5	24.3	25	25.9	26.8	27.8	28.8	29.8	31.2	32.3	32.7	33.2	34	34.7	36.4	38.2	39.6	41.4	41.7	41.8	28°C
	50	100	23	23.5	24.3	24.9	25.8	26.7	27.7	28.6	29.6	30.9	32.2	32.6	33.1	33.8	34.5	36.7	37.9	39.3	41.3	41.7	41.8	
	55	110	23	24.2	24.7	25.6	26.5	27.4	28.3	29.2	30.5	31.7	32.2	32.8	33.5	34.3	36.-	37.6	39.-	41.2	41.3	41.7	41.8	
	60	120	23	23.4	24.2	24.6	25.5	26.3	27.7	28	28.8	30	31.3	32.2	32.6	33.2	34	35.6	37.2	38.6	41.-	41.7	41.8	28°C
	65	130	23	23.4	24.1	24.5	25.4	26.1	27	27.8	28.7	29.9	31.1	32.2	32.6	33.2	33.9	33.4	37.1	38.5	40.9	41.7	41.8	
	66.75	133.5	N. II	One	side	frozen	on	top	≈	0.5	in³													
	70	140	23	23.6	24.2	24.7	25.5	26.2	27.-	27.8	28.6	29.7	31	32.1	32.5	33.1	33.8	35.2	36.9	38.2	40.8	41.7	41.8	
	75	150	23	23.5	24.1	24.5	25.5	26.2	27	27.7	28.5	29.7	30.8	31.9	32.3	33.0	33.75	35.2	36.7	38	40.6	41.7	41.8	28°C
	80	160	23	23.6	24.1	24.5	25.4	26.1	26.8	27.6	28.3	29.3	30.5	31.6	32.1	32.7	33.5	35.0	36.6	37.9	40.5	41.6	41.8	
	85	170	23	23.6	24.2	24.5	25.3	26.-	26.7	27.5	28.2	29.1	30.2	31.4	32.-	32.6	33.3	34.7	36.4	37.7	40.4	41.7	41.8	
	90	180	23	23.6	24.1	24.5	25.3	25.9	26.7	27.3	28.1	29.-	30.1	31.2	32.2	32.7	33.2	34.6	36.2	37.5	40.2	41.6	41.8	
	95	190	23	23.5	24	24.4	25.2	25.8	26.6	27.2	28	29.-	30	31.1	32.1	33.2	34.6	36	37.3	40.1	41.6	41.6	41.7	

FIGURE 15

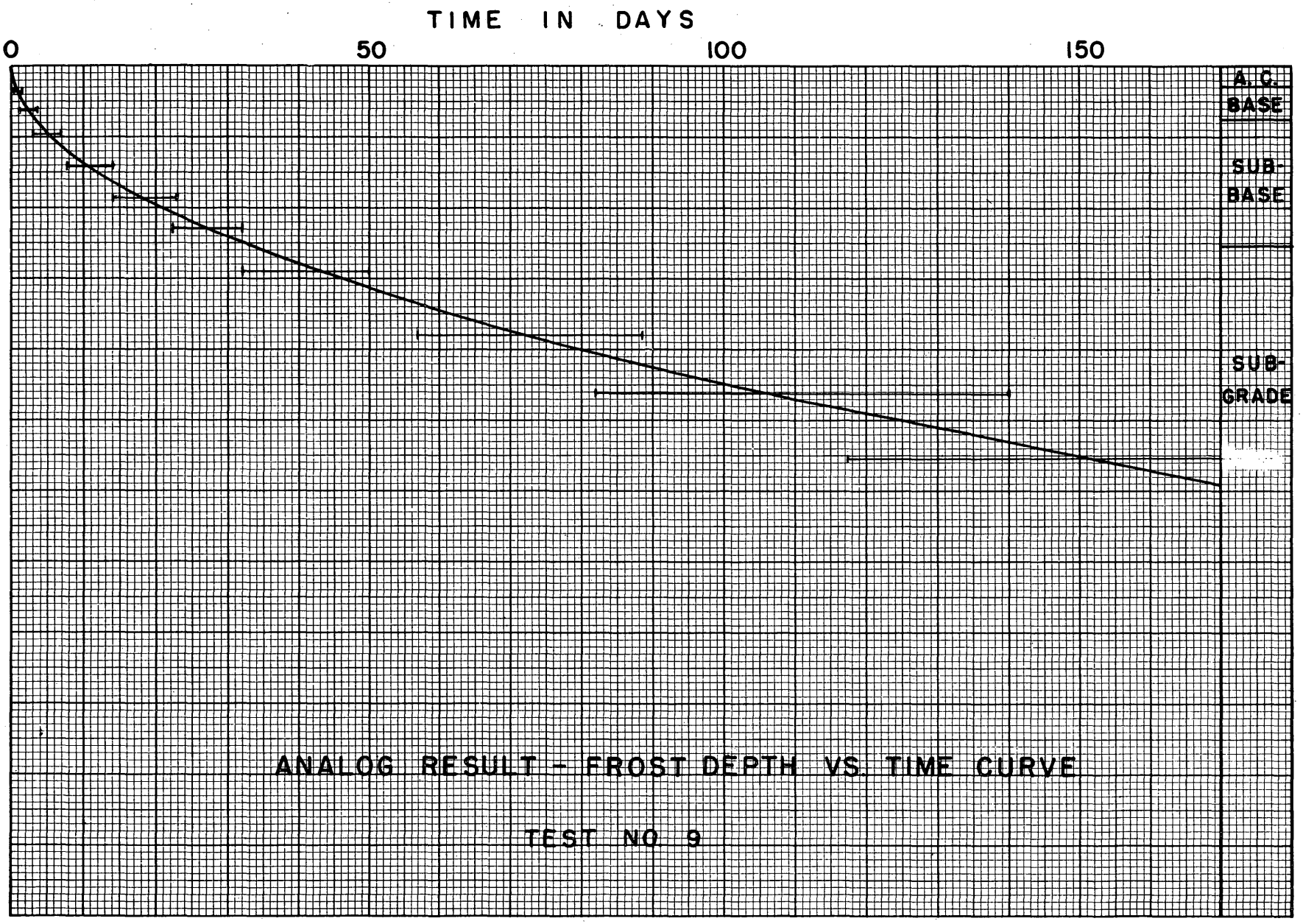

TIME IN DAYS
0
50
100
150
DEPTH IN FEET
0
1
2
3
4
5
6
7
8
A.C. BASE
SUB-BASE
SUB-GRADE
ANALOG RESULT - FROST DEPTH VS. TIME CURVE
TEST NO 9

FIGURE 16

Appendix A

UPPER LIMITS OF LAMINAR FLOW IN CAPILLARIES

From considerations of laminar flow through openings of any geometric cross-section (18):

$$Q = C_s \frac{\gamma_w R_H^2}{\mu} i a \qquad \text{- - - - (A-1)}$$

where:

Q, rate of flow in cu. cm. per sec.

γ_w, unit weight of water in g. per cu. cm.

R_H, hydraulic radius in cm.

μ, viscosity of water in g. sec. per sq. cm.

i, hydraulic gradient (dimensionless)

a, cross-section area in sq. cm.

and

C_s, a shape constant

= 1/2 for a circular capillary tube

= 1/3 for closely spaced parallel plates.

In tubes of circular cross-section, laminar flow will occur if:

$$\text{Reynolds number, } R_e = \frac{v\, d\, \gamma_w}{\mu\, g} < 2000 \qquad \text{- - - - (A-2)}$$

where:

v, velocity in cm. per sec.

d, tube diameter in cm.

At the upper limit of laminar flow, $R_e = 2000$, the relationship between hydraulic gradient and tube diameter becomes:

$$id = \frac{2000\ g}{C_s} \left(\frac{\mu}{\gamma_w R_H}\right)^2 \qquad ----(A-3)$$

Since:

$$R_H = \frac{d}{4} \text{ and } C_s = \frac{1}{2}$$

then:

$$id^3 = 4000\ g \left(\frac{4\mu}{\gamma_w}\right)^2$$

At a temperature of 70°F, $\mu = 10^{-5}$ and therefore:

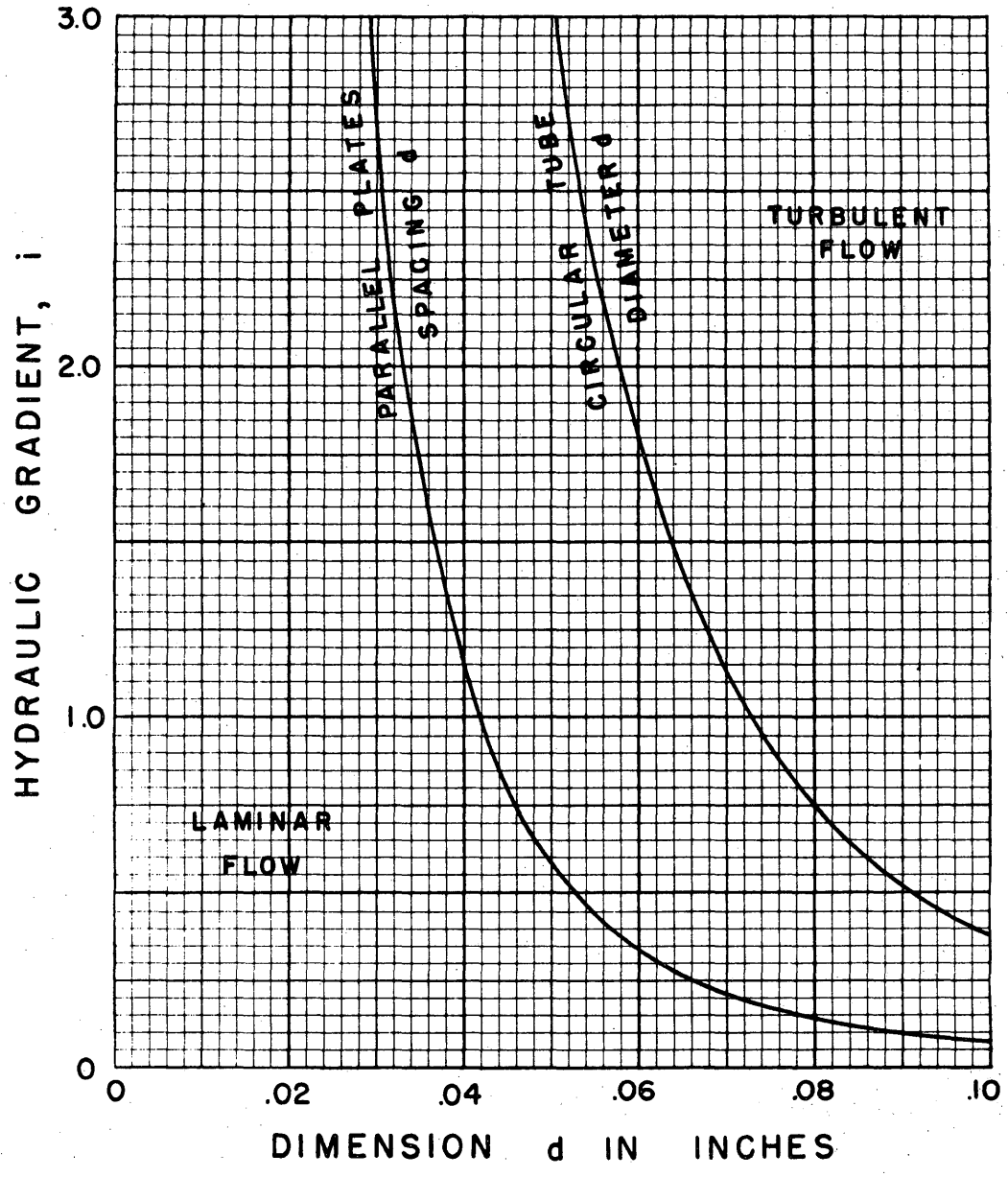
$$id^3 = 6.31 \times 10^{-3}\ cm.^3$$

$$= 3.84 \times 10^{-4}\ in.^3 \qquad ----(A-4)$$

For closely spaced parallel plates the upper limit of laminar flow (19) occurs when $R_e = 1000$ where d is defined as the distance between plates. In this case:

$$id^3 = 0.722 \times 10^{-4}\ in.^3 \qquad ----(A-5)$$

Curves of i vs d are given in the accompanying Figure A-1. A curve for a rectangular cross-section will lie somewhere between the two curves, approaching that for circular tubes when the cross-section is square. These curves will serve as rough guides for determining limiting dimensions for resistor capillaries.



RELATIONSHIP BETWEEN
HYDRAULIC GRADIENT AND DIMENSION d
FOR UPPER LIMIT OF LAMINAR FLOW THROUGH CIRCULAR TUBES AND BETWEEN PARALLEL PLATES FOR WATER AT 70°F

FIGURE A-1

Appendix B

CALIBRATION OF HYDRAULIC RESISTORS

The calibration of a hydraulic resistor may involve an experimental determination of the resistor setting required to give a desired value of resistance R_a. If a calibration curve is desired, the experimental procedure seeks to determine the resistance at several resistor settings so that a curve may be drawn, such as that shown in Figure B-2a.

A number of experimental procedures are available for calibrating a resistor. The most useful procedure depends on the investigator, the objective of the calibration and the physical circumstances under which the resistor exists.

In all cases the temperature of the water must be measured at the time of the calibration and all resistance determinations which are to be compared must be reduced to a common temperature by the following equation:

$$\frac{R_{a1}}{R_{a2}} = \frac{\mu_1}{\mu_2}$$

where μ is the viscosity of water.

B-01 Constant Head Test. -

A test setup for determining the hydraulic resistance by the constant head method is shown diagrammatically in Figure B-1a. The procedure consists simply of setting the resistor at the desired position, raising the headwater reservoir to give the required head loss $h_1 - h_2$, and determining the rate Q, at which water flows through the resistor. Thus:

$$R_a = \frac{h_1 - h_2}{Q}$$

It is generally felt that one of the following procedures, which make use of a falling head, represents a more desirable approach.

B-02 Falling Head Tests. -

Two falling head test procedures for calibrating hydraulic resistors are described below. These have the advantage that the resistor can be calibrated "in place" in the computer making use of a volumetric heat standpipe already connected to the resistor. In both cases the general setup for the test is shown in Figure B-1b. The resistor is connected to a standpipe on one side and to a reservoir which is suspended on a calibrated spring on the other. The reservoir has a cross-sectional area such that a rise in the water level produces an elongation of the spring of the same magnitude so that the water level remains at the same absolute elevation. A graduated scale is attached to the standpipe to facilitate the falling head reading. As an alternate to using the reservoir and spring combination, the tailwater reservoir can be any container such as the programming reservoir in the computer which can be easily adjusted to keep the tailwater elevation constant.

Water flows through the resistor, when the stopcock is opened, under a variable head h. We may write the following differential equation by continuity:

$$-\frac{dh}{dt}A = \frac{h}{R_a} \qquad \text{- - - - (B-1)}$$

where:

A, cross-sectional area of standpipe

t, time

Rearranging and integrating for the limits:

$t = 0$ when $h = h_o$

$t = t_1$ when $h = h_1$

and solving for R_a:

$$R_a = \frac{t_1}{A \ln \frac{h_o}{h_1}} \qquad \text{- - - - (B-2)}$$

or:

$$R_a = \frac{t_1}{A \cdot 2.3 \log \frac{h_o}{h_1}} \qquad \text{- - - - (B-3)}$$

(a) Falling head test with single observation. - It is frequently convenient to let

$$h_1 = \frac{h_o}{2}$$

in which case Equation (B-3) becomes:

$$R_a = 1.445 \frac{t_1}{A}$$

While this procedure is very quick, the result depends on one observation only. Indeed, it is possible to repeat the test but the following procedure offers additional advantages.

(b) Falling head test with multiple observations. - Equation (B-3) shows that if R_a is constant, t_1 is proportional to log h_o/h_1. It follows then that a plot of t_1 to a natural scale vs. h_o/h_1 to a logarithmic scale will yield a straight line for laminar flow.

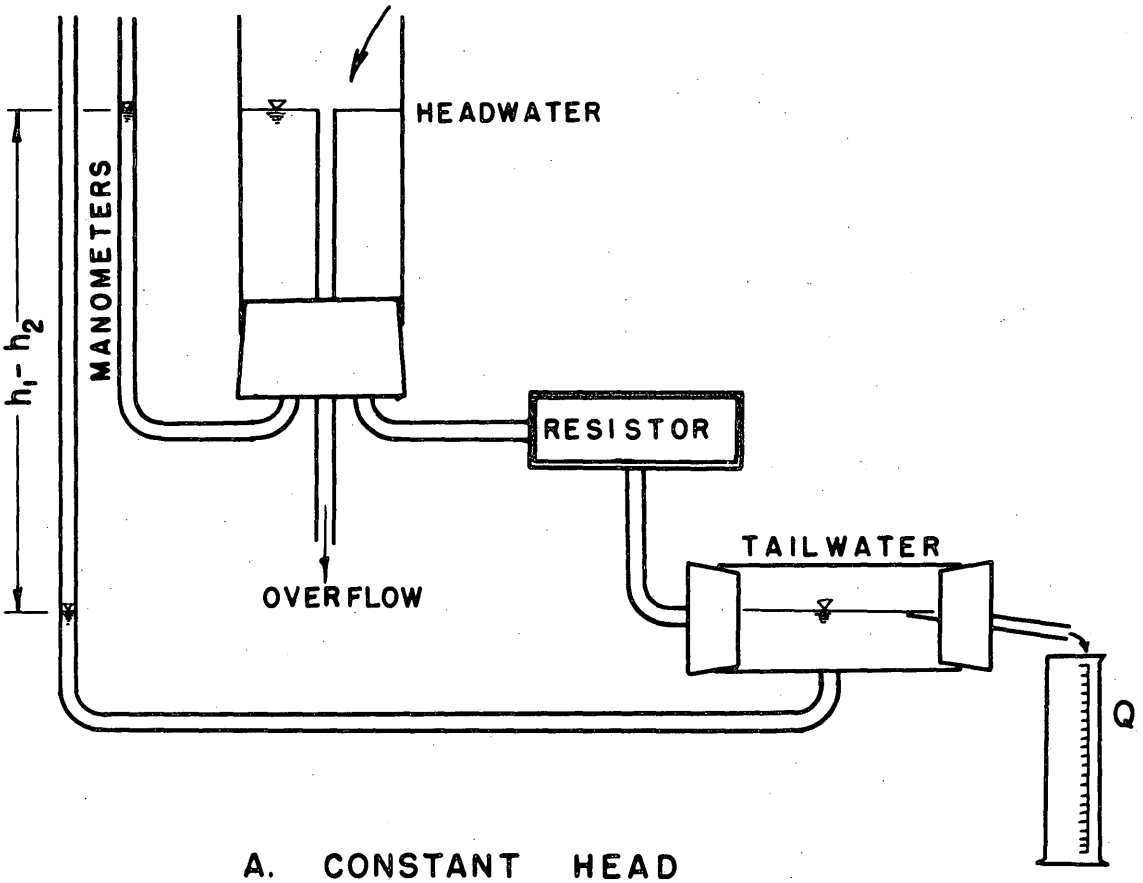
This latter fact is utilized where the falling head is recorded as a function of elapsed time. The observations are plotted on semi-logarithmic paper and the resistance is computed from Equation (B-3). The term

$$\frac{t_1}{\log \frac{h_o}{h_1}}$$

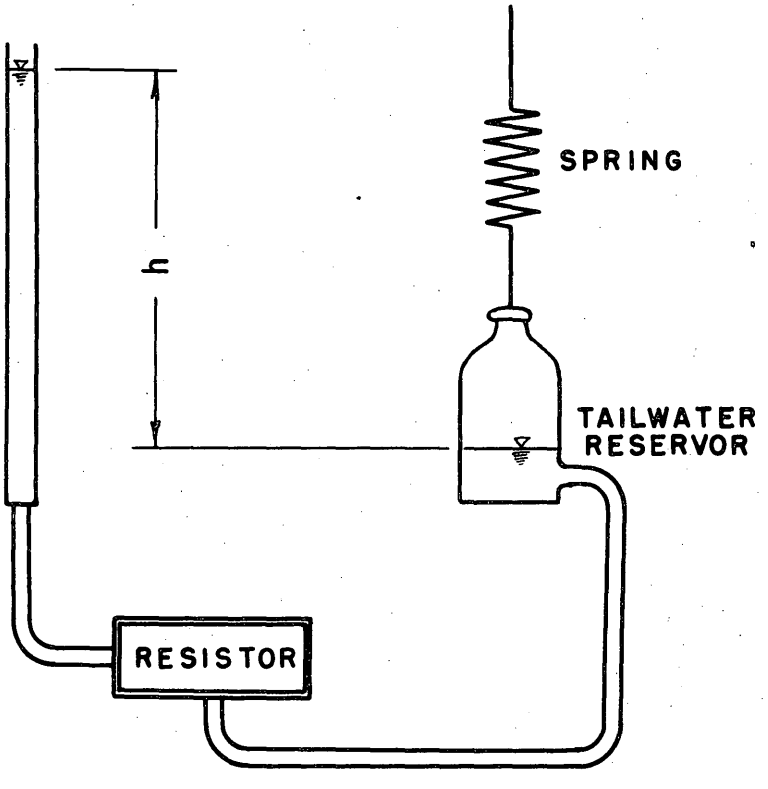
is the time intercept over one log cycle. An example is shown in Figure (B-2b).

This procedure is more elaborate than that based on a single observation of time. However, since the result is based on a series of observations it is far more dependable than the former. Moreover, the test can be used to check for laminar flow through the resistor. For a given capillary length there exists a critical gradient above which the laminar nature of the flow breaks down and becomes turbulent. In this case the semi-log curve deviates from a straight line and R is no longer independent of head.

Since the resistors normally operate at heads of less than one inch, the calibration should be carried out at heads of no more than 3 or 4 inches in order to minimize any small dependence of resistance on head.



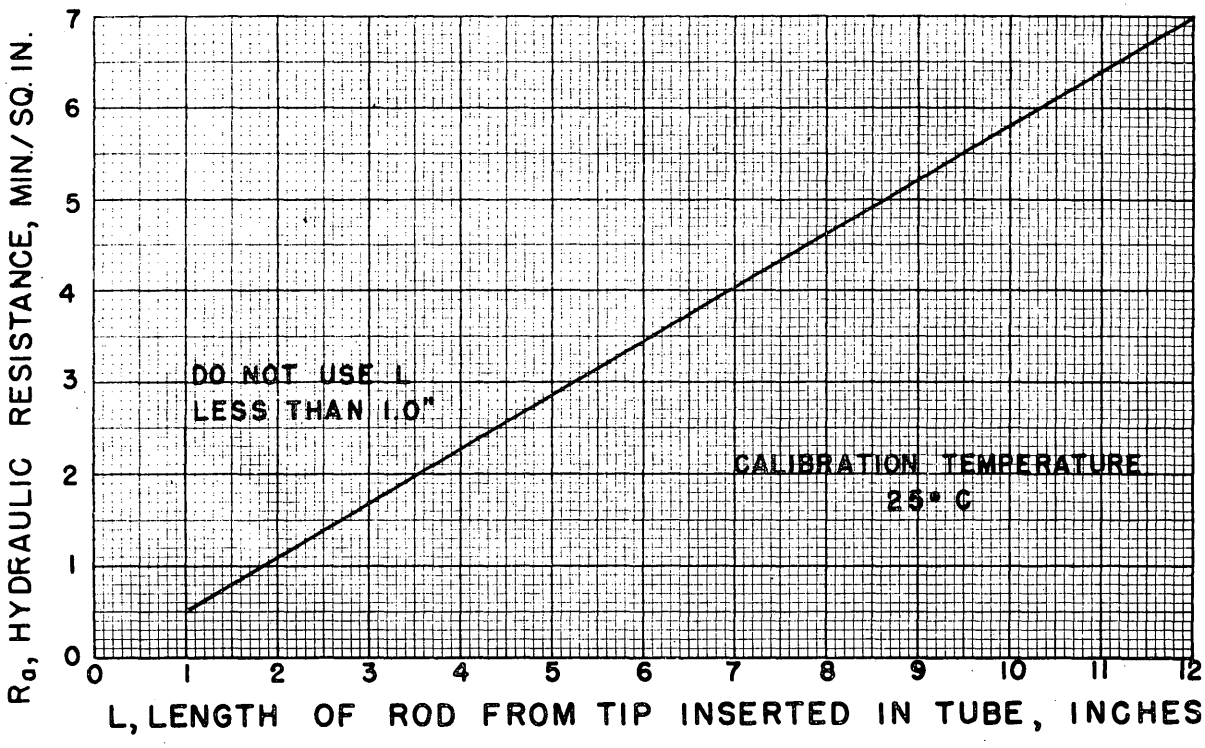
A. CONSTANT HEAD



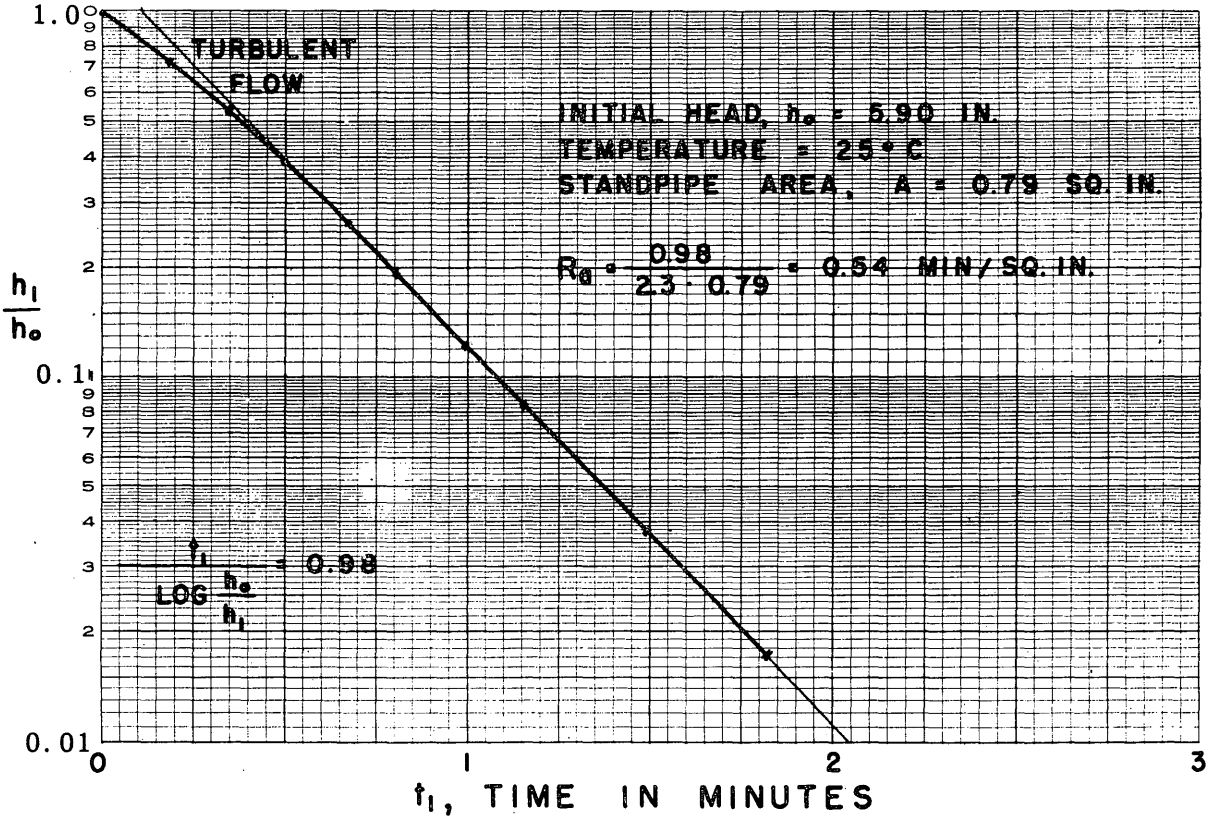
B. FALLING HEAD

SCHEMATIC DRAWINGS OF RESISTANCE CALIBRATION METHODS

FIGURE B-1



A. APPROXIMATE CALIBRATION CURVE FOR RESISTOR TYPE B



B. FALLING HEAD TEST WITH MULTIPLE OBSERVATIONS

EXAMPLES OF RESISTOR CALIBRATION CHARTS

FIGURE B-2